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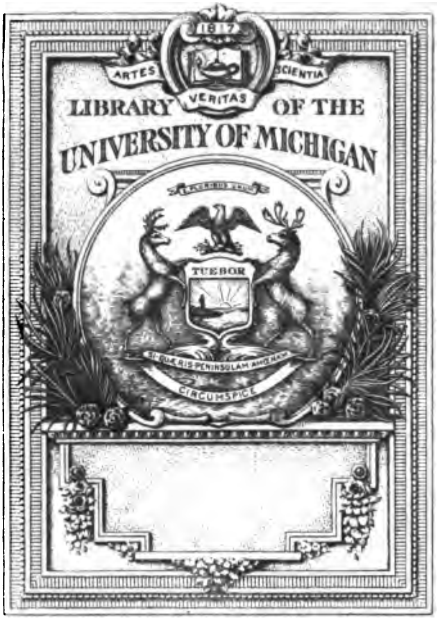
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Essentials

OF

Crystallography

GEORGE WAHR, Publisher
Ann Arbor, Mich.
1906

ERRATA.

Page 26, last line, for $78^\circ\ 31'\ 44''$ read $70^\circ\ 31'\ 44''$.

Page 35, line 8, for $\pm\left[\frac{a : ma : a\infty}{2}\right]$ read $\pm\left[\frac{a : ma : \infty a}{2}\right]$.

Page 37, line 6 from the bottom, for effected read affected.

Page 42, last line, for $\lambda = + \frac{402}{2} l$ read $\lambda = + \frac{402}{4} l$.

Page 45, line 3, for planes read plane.

Page 47, line 14, for triangle read triangles.

Essentials

OF

Crystallography

By

EDWARD HENRY KRAUS, Ph.D.

Junior Professor of Mineralogy in the
University of Michigan

WITH 427 FIGURES

GEORGE WAHR, Publisher
Ann Arbor, Mich.
1906

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PREFACE.

This text is intended for beginners and aims to present the essential features of geometrical crystallography from a standpoint which combines the ideas of symmetry with those of holohedrism, hemihedrism, etc. All forms possible in the thirty-two classes of symmetry are discussed even though representatives of several of the classes have not as yet been observed among minerals or artificial salts. In each system, however, the important classes are indicated so that the book may readily be used for abridged courses.

No attempt has been made to discuss the measurement and projection of crystals inasmuch as they involve a somewhat comprehensive knowledge of forms and, hence, cannot be treated adequately in an introductory course. For a similar reason very little reference has been made to the various structural theories of crystals. However, the bibliography of the more important works, which have been published in English and German during the past thirty years, will prove useful to those who may desire to pursue the study of crystallography further than the scope of this text permits.

Free use has been made of the various standard texts on crystallography, but especial acknowledgments are due von Groth, Linck, Bruhns, Tschermak, and E. S. Dana. A very large majority of the figures have been drawn especially for this text, the others, however, have been taken from various sources.

I am indebted to Mr. W. F. Hunt, Instructor in Mineralogy, also to Messrs. I. D. Scott and C. W. Cook, Assistants in Mineralogy in the University of Michigan, for aid in the reading of the proof.

EDWARD H. KRAUS.

Mineralogical Laboratory, University of Michigan,
Ann Arbor, Mich., June, 1906.

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INTRODUCTION.

Crystals and Crystalline Structure. Nearly all homogeneous substances possessing a definite chemical composition, when solidifying from either a solution, state of fusion or vapor, attempt to crystallize, that is, to assume certain characteristic forms. If the process of solidification is slow enough and uninterrupted, regular forms, bounded by plane surfaces, usually result. These regular polyhedral forms are termed *crystals.* If, however, the solidification is so rapid that the substance cannot assume well defined forms, bounded by plane surfaces, the solid mass, which results, is said to be *crystalline.* Crystalline substances consist of aggregations of crystals, which have been hindered in their development. Their outline and orientation are, hence, irrregular. Figure 1 shows a well developed crystal of calcite ($CaCO_3$), while figure 2[1] represents a cross-section through a crystalline aggregate of the same substance. Here, no definite outline is to be recognized on the component parts of the mass.

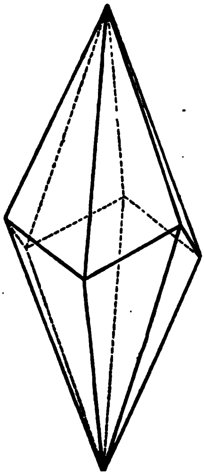


Fig. 1.

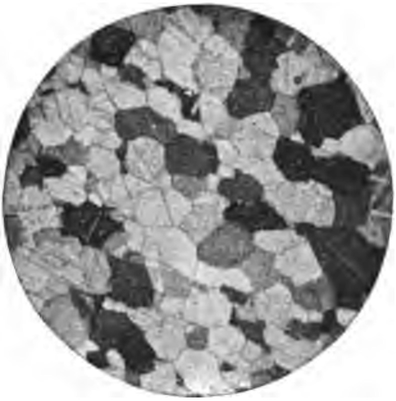


Fig. 2.

Amorphous Structure. Those substances, however, which do not attempt to crystallize, when solidifying, are termed *amorphous.*

---

1) After Weinschenk.

They are without form. Opal ($SiO_2 . x H_2O$), for example, never occurs in well defined crystals or in crystalline masses.

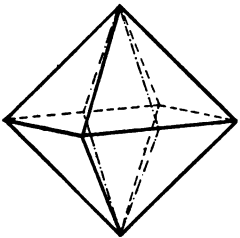
Some Properties of Crystallized and Amorphous Substances. There are many interesting features to be noted in the study of crystallized and amorphous substances. The attempt at crystallization and the assuming of regular polyhedral forms, which is characteristic of bodies belonging to the first class, are only some of the expressions of the nature of such substances. The various physical properties, hardness, elasticity, cohesion, transmission of heat and light, usually vary in such crystallized substances with the direction according to fixed laws. This is not the case with amorphous substances. Hence, we may define *a crystal as a solid body, which is bounded by natural plane surfaces and whose form is dependent upon its physical and chemical properties.*

Crystallography. This science treats of the various properties of crystals and crystallized bodies. It may be subdivided as follows:

1. *Geometrical Crystallography.*
2. *Physical Crystallography.*
3. *Chemical Crystallography.*

Geometrical crystallography, as the term implies, describes the various forms occurring upon crystals. The relationships existing between the crystal form and the physical and chemical properties of crystals are the subjects of discussion of the second and third subdivisions of this science, respectively. Only the essentials of the first subdivision—geometrical crystallography—will be treated in this text.

Constancy of Interfacial Angles. As indicated on page 1, crystals may, in general, result from solidification from a solution, state of fusion, or vapor. Let us suppose that some ammonium alum, $(NH_4)_2Al_2(SO_4)_4.24H_2O$, has been dissolved in water and the solution allowed to evaporate slowly. As the alum begins to crystallize, it will be noticed that the crystals are, for the most part, bounded by eight plane surfaces. If these surfaces are all of the same size, that is, have been equally developed, the crystals will possess an outline as represented by figure 3. Such a form is termed an *octahedron.* The octahedron is bounded by eight equilateral triangles. The angles between any two adjoining surfaces or *faces*, as they are often called, is the same, namely, 109° 28¼′. On most of the crystals, however, it will be seen that the various faces have been developed unequally, giving rise to the forms illustrated by figures 4 and 5. Similar cross-sections through these forms are shown in figures 6, 7, and 8, and it is readily seen that, although the size of the faces and, hence, the resulting shapes have been materially changed, the angle between the adjoining faces has remained the same, namely, 109° 28¼′. Such forms of the octahedron are said to be *misshapen* or *distorted*. *Distortion* is quite common on all crystals regardless of their chemical composition.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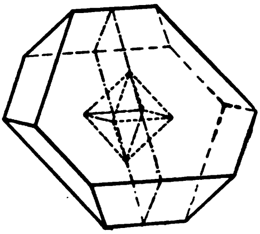
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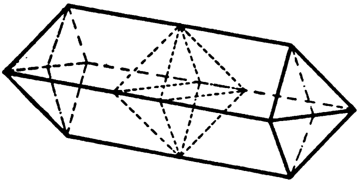
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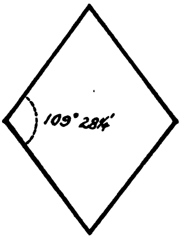

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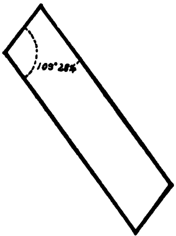

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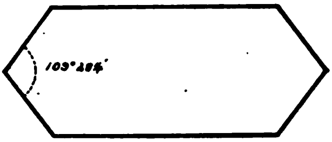

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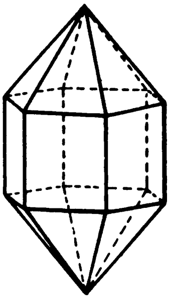
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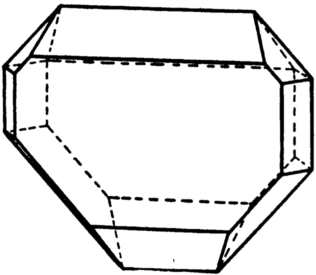
Fig. 10.

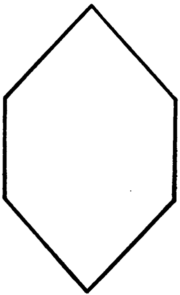
Fig. 11.

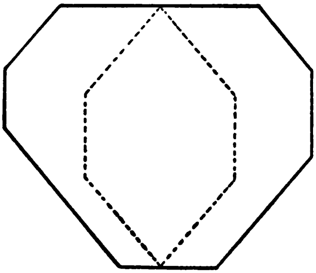
Fig. 12.

It was the Danish physician and natural

scientist, Nicolas Steno, who in 1669 first showed that the angles between similar faces on crystals of quartz remain constant regardless of their development. Figures 9 and 10 represent two crystals of quartz with similar cross-sections (figures 11 and 12). Further observations, however, showed that this applies not only to quartz but to all crystallized substances and, hence, we may state the law as follows: *Measured at the same temperature, similar angles on crystals of the same substance remain constant regardless of the size or shape of the crystal.*

Crystal Habit. The various shapes of crystals, resulting from the unequal development of their faces, are oftentimes called their *habits.* Figures 3, 4, and 5 show some of the habits assumed by alum crystals. In figure 3, the eight faces are about equally developed and this may be termed the *octahedral* habit. The *tabular* habit, figure 4, is due to the predominance of two parallel faces. Figure 5 shows four parallel faces predominating, and the resulting form is the *prismatic* habit.

Crystallographic Axes. Inasmuch as the crystal form of any substance is dependent upon its physical and chemical properties, page 2, it necessarily follows that an almost infinite variety of forms is possible. In order, however, to study these forms and define the position of the faces occurring on them advantageously, straight lines of definite lengths are assumed to pass through the ideal center of each crystal. These lines are the *crystallographic axes.* Their intersection forms the *axial cross.* Figure 13 shows the octahedron referred to its three crystal axes. In this case the axes are of equal lengths and termed *a* axes. The extremities of the axes are differentiated by the use of the plus and minus signs, as shown in figure 13.

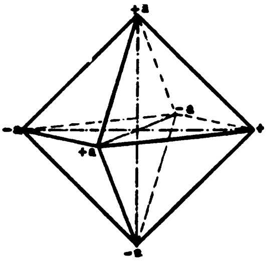

Fig. 13.

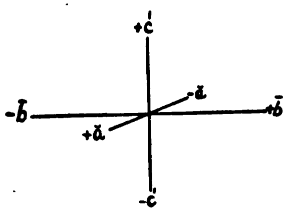

Fig. 14.

If the axes are of unequal lengths, the one extending from front to rear is termed the *a* axis, the one from right to left the *b*, while the vertical axis is called the c axis. This is illustrated by figure 14. The axes are always referred to in the following order, viz: a, b, c.

Crystal Systems. Although a great variety of crystal forms is possible, it has been shown in many ways that all forms may be classified into six large groups, called *crystal systems*. In the grouping of crystal forms into systems, we are aided by the crystallographic axes. The systems may be differentiated by means of the axes as follows:

1. **Cubic System.** Three axes, all of equal lengths, intersect at right angles. The axes are designated by the letters, a, a, a.

2. **Hexagonal System.** Four axes, three of which are equal and in a horizontal plane intersecting at angles of 60°. These three axes are often termed the *lateral* or *secondary* axes, and are designated by a, a, a. Perpendicular to the plane of the lateral axes is the vertical axis, which may be longer or shorter than the a axes. This fourth axis is called the *principal* or c axis.

3. **Tetragonal System.** Three axes, two of which are equal, horizontal, and perpendicular to each other. The vertical, c, axis is at right angles to and either longer or shorter than the horizontal or *lateral*, a, axes.

4. **Orthorhombic System.** Three axes of unequal lengths intersect at right angles. These axes are designated by $\breve{a}$, $\bar{b}$, $\acute{c}$, as shown in figure 14.

5. **Monoclinic System.** Three axes, all unequal, two of which (a', $\acute{c}$) intersect at an oblique angle, the third axis ($\bar{b}$) being perpendicular to these two.

6. **Triclinic System.** Three axes ($\breve{a}$, $\bar{b}$, $\acute{c}$) are all unequal and intersect at oblique angles.

Parameters and Parametral Ratio. In order to determine the position of a face on a crystal, it must be referred to the crystallographic axes. Figure 15 shows an axial cross of the orthorhombic system. The axes, a, b, c, are, therefore, unequal and perpendicular to each other. The plane ABC cuts the three axes at the points A, B, and C, hence, at the distance OA=a, OB=b, OC=c, from the center, O. These distances, OA, OB, and OC, are known as the *parameters* and the ratio, OA : OB : OC, as the *parametral ratio* of the plane ABC. This ratio may, however, be abbreviated to $a : b : c$.

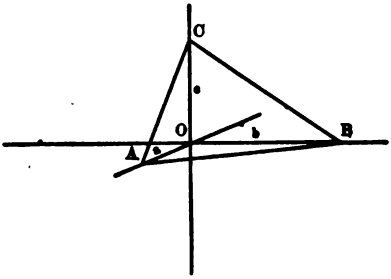

Fig. 15.

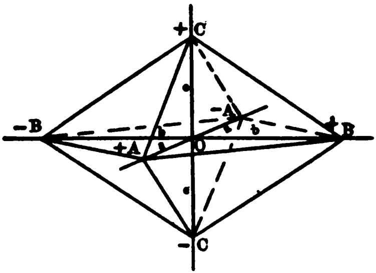

Fig. 16.

There are, however, seven other planes possible about this axial cross which possess parameters of the same lengths as those of the plane ABC, figure 16. The simplified ratios of these planes are:

$a : -b : c$
$a : b : -c$
$a : -b : -c$
$-a : b : c$
$-a : -b : c$
$-a : b : -c$
$-a : -b : -c$

These eight planes, which are similarly located with respect to the crystallographic axes, constitute a *crystal form*, and may be represented by the general ratio $(a : b : c)$. The number of faces in a crystal form depends, moreover, not only upon the intercepts or parameters but also upon the elements of symmetry possessed by the crystal, see page 12. Those forms, which enclose space, are called *closed* forms. Figure 16 is such a form. Those, however, which do not enclose space on all sides, as shown in figure 17, are termed *open* forms.

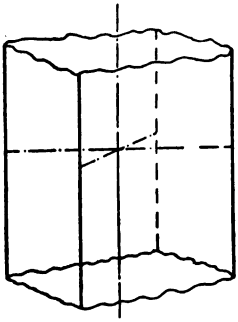

Fig. 17.

Fundamental Form. In figure 18, the enclosed form possesses the general ratio, $a : b : c$. The face ABM, however, has the parametral ratio, oA : oB : oM, where $oA = a$, $oB = b$, and $oM = 3oC = 3c$. Hence, this ratio may be written $a : b : 3c$. But, as in the previous case, this ratio represents a form consisting of eight faces as shown in the figure. That form, the parameters of which are selected as the unit lengths of the crystallographic axes, is known as the *unit* or *fundamental* form. In figure 18 the inner pyramid is a so-called *unit*, whereas the other one is a *modified* pyramid.

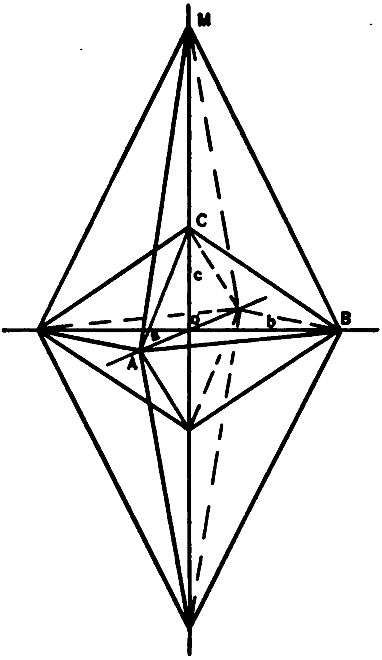

Fig. 18.

Combinations. Several different forms may occur simultaneously upon a crystal, giving rise to a *combination*. Figure 19 shows a combination of two pyramids observed on sulphur; p = $a : b : c$ (unit) and s $= a : b : \frac{1}{3}c$ (modified). Figure 20 shows the two forms, o $= a : a : a$, and h $= a : \infty a : \infty a$, see page 19.

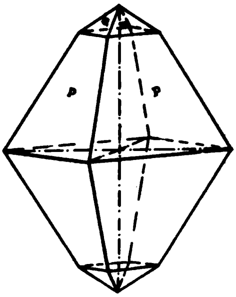

Fig. 19.

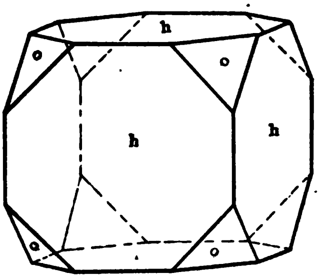

Fig. 20.

Axial Ratio. If the intercepts of a unit form cutting all three axes be expressed in figures, the intercept along the b axis being considered as unity, we obtain the *axial ratio*. In figure 19, which represents a crystal of sulphur, the axial ratio is:

$$a : b : c = .8131 : 1 : 1.9034.$$

Every crystallized substance has its own axial ratio. This is illustrated by the ratios of three minerals crystallizing in the orthorhombic system.

Aragonite, $CaCO_3$, $a : b : c = .6228 : 1 : .7204$
Anglesite, $PbSO_4$, $a : b : c = .7852 : 1 : 1.2894$
Topaz, $Al_2(F.OH)_2\ SiO_4$, $a : b : c = .5281 : 1 : .9442$

In the hexagonal and tetragonal systems, since the horizontal axes are all equal, *i. e.*, $a = b$, see page 5, the axial ratio is reduced to $a : c$; a now being unity. Thus, the axial ratio of zircon ($ZrSiO_4$) which is tetragonal, may be expressed as follows: $a : c = 1 : .6404$; that of quartz (SiO_2), hexagonal, by $a : c = 1 : 1.0999$. Obviously, in the cubic system, page 5, where all three axes are equal, this is unnecessary.

However, in the monoclinic and triclinic systems, where either one or more axes intersect obliquely, it is not only necessary to give the axial ratio but also to indicate the values of the angles between the crystallographic axes. For example, gypsum ($CaSO_4 . 2H_2O$) crystallizes in the monoclinic system and has the following axial ratio:

$$a' : \bar{b} : \acute{c} = .6896 : 1 : .4133$$

and the inclination of the a' axis to the $\acute{c}$ is 98° 58′. This angle is known as β, figure 21.

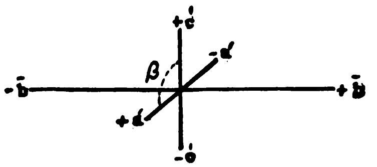

Fig. 21.

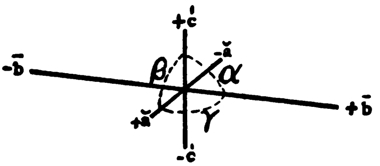

Fig. 22.

In the triclinic system, since all axes are inclined to each other, it is also necessary to know the value of the three angles, which are located as shown in figure 22, viz: $\bar{b} \wedge \acute{c} = \alpha$, $\breve{a} \wedge \acute{c} = \beta$, $\breve{a} \wedge \bar{b} = \gamma$. The axial ratio and the angles showing the inclination of the axes are termed the *elements of crystallization.*

The triclinic mineral albite ($NaAlSi_3O_8$) possesses the following elements of crystallization:

$$\breve{a} : \bar{b} : \acute{c} = .6330 : 1 : 5573.$$
$$\alpha = 94^\circ\ 5'$$
$$\beta = 116^\circ\ 27'$$
$$\gamma = 88^\circ\ 7'$$

If the angles between the crystallographic axes equal 90°, they are not indicated. Therefore, in the tetragonal, hexagonal, and orthorhombic systems, the axial ratios alone constitute the elements of crystallization, while in the cubic system, there are no unknown elements.

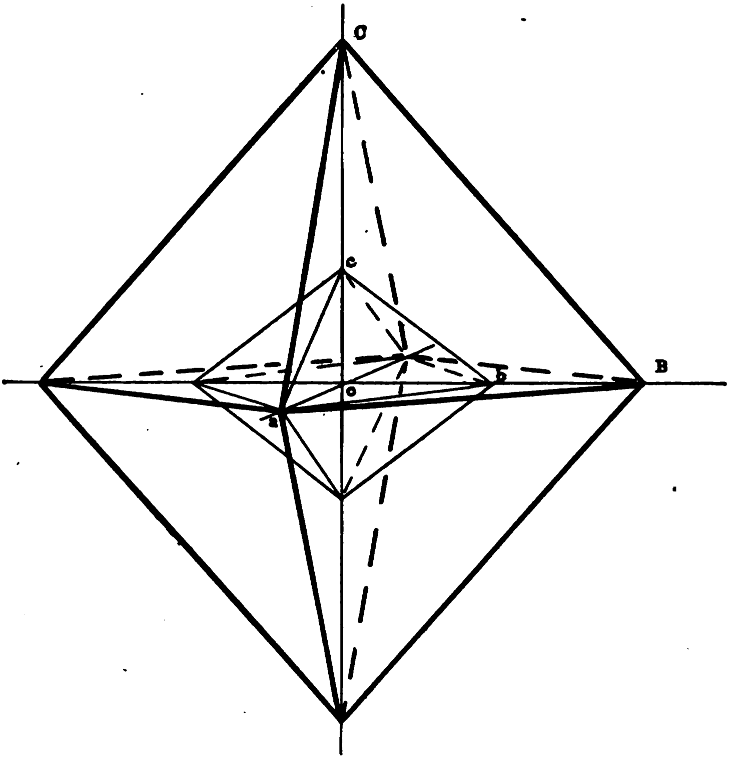

Fig. 23.

Rationality of Coefficients. The parametral ratio of any face may be expressed in general by $na : pb : mc$, where the coefficients n, p, m, are according to observation always rational. In figure 23, the inner pyramid is assumed to be the fundamental form, page 7, with the following value of the intercepts: oa = 1.256, ob = 1, oc = .752. But the coefficients n, p, m, are obviously all equal to unity. The ratio is, hence, $a : b : c$.

The outer pyramid, however, possesses the intercepts, oa = 1.256, oB = 2, oC = 2.256. These lengths, divided by the unit lengths of each axis, as indicated above, determine the values of n, p, and m for the outer pyramid, namely:

$$n = \frac{1.256}{1.256} = 1; \quad p = \frac{2}{1} = 2; \quad m = \frac{2.256}{.752} = 3.$$

These values of n, p, and m, are, therefore, rational. Such values as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{5}{3}$, or $\frac{2}{5}$ are also possible, but never 3.1416+, 2.6578+, $\sqrt{3}$, and so forth.

Symbols. The parametral ratio of the plane ABM, figure 18, may be written as follows:

$$n\breve{a} : p\bar{b} : m\dot{c}.$$

But since, in this case, $n = 1$, $p = 1$, $m = 3$, the ratio becomes:

$$\breve{a} : \bar{b} : 3\dot{c}.$$

If, however, the coefficients had the values $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{3}$, respectively, the ratio would then read:

$$\tfrac{1}{2}\breve{a} : \tfrac{2}{3}\bar{b} : \tfrac{4}{3}\dot{c}.$$

This, when expressed in terms of b, becomes:

$$\tfrac{3}{4}\breve{a} : \bar{b} : 2\dot{c}.$$

Hence, the ratio

$$na : b : mc$$

expresses the most general ratio or symbol for forms belonging to the orthorhombic, monoclinic, and triclinic systems. In the hexagonal and tetragonal systems, since the a and b axes are equal, this general symbol becomes,

$$a : na : mc.$$

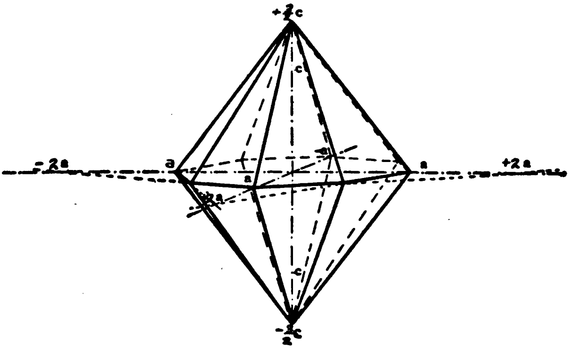

Fig. 24.

Figure 24 shows a form, the ditetragonal bipyramid, with the symbol $a : 2a : \frac{3}{2}c$. In the cubic system, all three axes are equal and the general symbol reads,

$$a : na : ma.$$

The ratio $a : \infty a : \infty a$, for example, symbolizes a form in the cubic system consisting of six faces, which cut one axis and extend parallel to the other two. Such a form is the cube, figure 25. The ratio $a : 2a : \infty a$ represents a form with twenty-four faces; each face cuts one axis at a unit's length, the second at twice that length, but extends parallel to the third axis. Figure 26 shows such a form, the tetrahexahedron. This system of crystallographic notation is known as the Weiss system after the inventor Prof. C. S. Weiss. These symbols are most readily understood and well adapted for beginners.

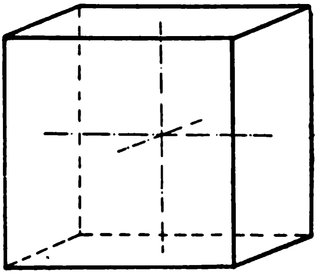
Fig. 25.

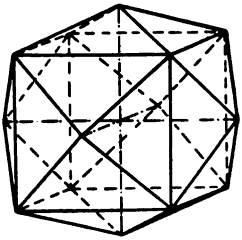
Fig. 26.

Naumann, Dana, and Miller have introduced modifications tending to shorten the symbols of Weiss. Since these shortened forms are employed quite extensively, it will be necessary to explain each briefly.

Symbols of Naumann and Dana. In the notation of Naumann, O and P, the initial letters of the words, octahedron and pyramid, respectively, are used as a basis. O is used for forms of the cubic system, P for all other systems. The coefficient m, referring to the vertical axis, is placed before and the other coefficient n, after one of these letters. For example, $n\breve{a} : \bar{b} : m\acute{c}$, becomes $m\mathrm{P}\breve{n}$, and $a : ma : \infty a$, $\infty \mathrm{O} m$.

Dana's notation is similar to that of Naumann. Dana, however, substitutes a short dash for the letters O and P, also i or I for ∞, the sign of infinity. Otherwise, the two systems are alike.

The following table shows several Weiss symbols with the corresponding Naumann and Dana modifications:

Weiss	Naumann	Dana
$\breve{a} : \bar{b} : 3\acute{c}$	$3\mathrm{P}\breve{\ }$	$3\ \breve{\ }$
$\frac{3}{4}\breve{a} : \bar{b} : 2\acute{c}$	$2\mathrm{P}\breve{\tfrac{3}{4}}$	$2-\breve{\tfrac{3}{4}}$
$a : 2a : \frac{3}{2}c$	$\frac{3}{2}\mathrm{P}2$	$\frac{3}{2}-2$
$a : \infty a : \infty a$	$\infty\mathrm{O}\infty$	$i-i$
$a : 2a : \infty a$	$\infty\mathrm{O}2$	$i-2$
$a : na : ma$	$m\mathrm{O}n$	$m-n$

Miller's System. In this system, as is also the case with the notations of Naumann and Dana, the letters referring to the various crystallographic axes are not indicated. The values given being understood as referring to the *a*, *b*, and *c* axes respectively, page 5. The reciprocals of the Weiss parameters are reduced to the lowest common denominator, the numerators then constitute the Miller symbols, called *indices*. For example, the reciprocals of the Weiss parameters $2a : b : 3c$ would be $\frac{1}{2}$, $\frac{1}{1}$, $\frac{1}{3}$. These, reduced to the lowest common denominator, are $\frac{3}{6}$, $\frac{6}{6}$, $\frac{2}{6}$. Hence, 3 6 2 constitute the corresponding Miller indices. These are read *three*, *six*, *two*.

A number of examples will make this system of notation clear. Thus, $a : \infty b : \infty c$, becomes 100; $2a : b : 5c$, 5.10.2; $a : a : 3c$, 331; $a : \infty a : 2c$, 201, and so forth. The Miller indices corresponding to the general ratios $a : na : ma$ and $na : b : mc$ are written *hkl*. The Miller indices are important because of their almost universal application in crystallographic investigations. The transformation of the Naumann or Dana symbols to those of Miller should, after the foregoing explanations, present no difficulty whatever.

Elements of Symmetry. The laws of symmetry find expression upon a crystal in the distribution of similar angles and faces. The presence, therefore, of planes, axes, or a center of symmetry — these are the *elements of symmetry* — is of great importance for the correct classification of a crystal.

Planes of Symmetry. Any plane, which passes through the center of a crystal and divides it into two symmetrical parts, the one-half being the *mirror-image* of the other, is a *plane of symmetry*. Figure 27 shows a crystal of gypsum ($CaSO_4 . 2H_2O$) with its one plane of symmetry. Every plane of symmetry is parallel to some face, which is either *present* or *possible* upon the crystal. In figure 27 it is parallel to the face b.

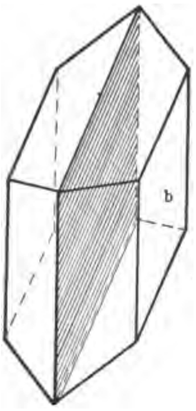

Fig. 27.

It is sometimes convenient to subdivide the planes of symmetry into *principal* and *secondary* or *common* planes, according to whether they possess two or more equivalent and interchangeable directions or not. Figure 28 illustrates a crystal of the tetragonal system with five planes of symmetry. The horizontal plane *c* is the *principal*, the vertical planes the *secondary* planes of symmetry.

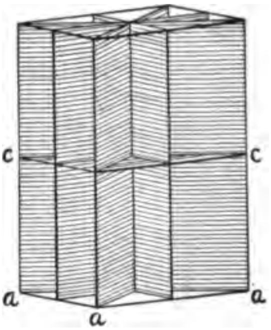

Fig. 28.

Axes of Symmetry. The line, about which a crystal may be revolved as an axis so that after a definite angular revolution the crystal assumes exactly the same position in space which it originally had, is termed an *axis of symmetry*. Depending upon the rotation necessary, only four types of axes of symmetry are from the standpoint of crystallography possible.[1]

a) Those axes, about which the original position is reassumed after a revolution of 60°, are said to be axes of *hexagonal*, *six-fold*, or *six-count*[2] symmetry. Such axes may be indicated by the symbol ⬢. Figure 29 shows such an axis.

b) If the original position is regained after the crystal is revolved through 90°, the axis is termed a *tetragonal*, *four-count*, or *four-fold* axis of symmetry. These axes are represented by ■, as illustrated in figure 30.

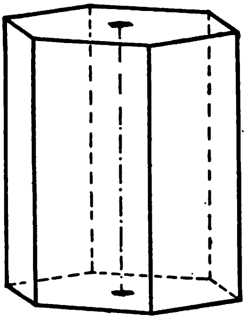

Fig. 29.

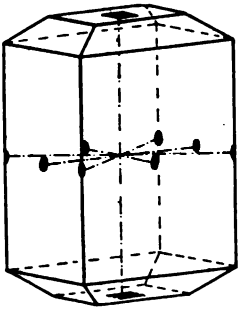

Fig. 30.

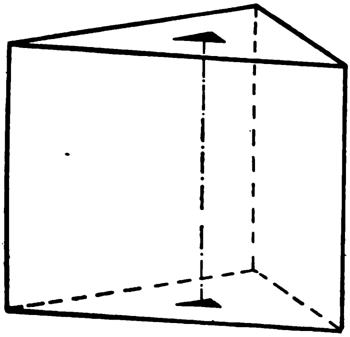

Fig. 31.

c) Axes requiring an angular revolution of 120° are *trigonal*, *three-fold*, or *three-count* axes of symmetry and may be symbolized by ▲. Figure 31 illustrates this type of axis.

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[1] For proof, see Groth's Physikalische Krystallographie, 4te Auflage, 1905, 321; also Viola's Grundzüge der Kristallographie, 1904, 251.

[2] Because in a complete revolution of 360° the position is reassumed six times.

d) A *binary*, *two-fold*, or *two-count* axis necessitates a revolution through 180°. These are indicated by ● in figure 30.

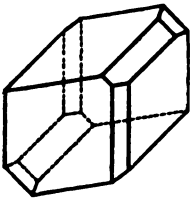

Fig 32.

Center of Symmetry. That point within a crystal through which straight lines may be drawn, so that on either side of and at the same distance from it, similar portions of the crystal (faces, edges, angles, and so forth) are encountered, is a *center of symmetry*. Figure 32 has a center of symmetry — the other elements of symmetry are lacking.

Angular Position of Faces. Since crystals are oftentimes *misshapen* or *distorted*, page 3, it follows that the elements of symmetry are not always readily recognized. The *angular position* of the faces in respect to these elements is the essential feature, and not their distance or relative size. Figure 33 shows an ideal crystal of augite. Here, the presence of a plane of symmetry, an axis, and a center of symmetry is obvious. Figure 34 shows a distorted crystal of the same mineral, possessing however exactly the same elements of symmetry, because the angular position of the faces is the same as in figure 33.

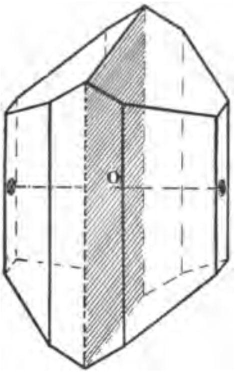

Fig. 33.

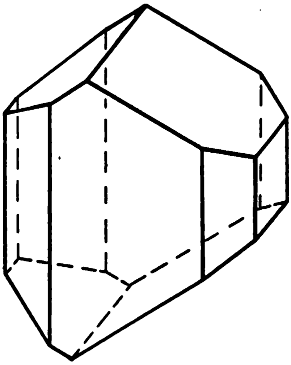

Fig. 34.

Classes of Symmetry. Depending upon the elements of symmetry present, crystals may be divided into *thirty-two* distinct groups, called *classes of symmetry*.[1] Only forms which belong to the same class can occur in combination with each other. A *crystal system*, however, includes all those classes of symmetry which can be referred to the same type of crystallographic axes, page 5. The various elements of symmetry and, wherever possible, an important representative are given for each of the thirty-two classes in the tabular classification on page 150.

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[1] Also termed *classes of crystals*.

Holohedrism. Figure 35 shows a form of the cubic system. The face a b c has the parametral ratio $a : \frac{3}{2}a : 3a$. Forty-seven other faces, having this same ratio, are however also possible and, when present, give rise to the *hexoctahedron*, as the complete form is called. These forty-eight faces are all equal, scalene triangles. Such forms, which possess all the faces possible having the same ratio, are called *holohedral* forms.

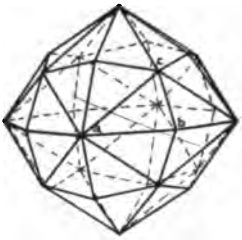

Fig. 35.

Hemihedrism. If, however, one-half of the faces of the hexoctahedron be suppressed and the other half be allowed to expand, as shown in figure 36, the *diploid* with half of the faces possessed by the hexoctahedron, namely twenty-four, results. Forms of this character are said to be *hemihedral*.

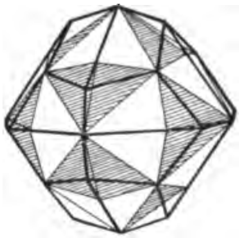

Fig 36.

Tetartohedrism. Again, if only one quarter of the faces expand, all others being suppressed, the hexoctahedron then yields the *tetrahedral pentagonal dodecahedron* with but twelve faces, figure 37. This is a *tetartohedral* form.

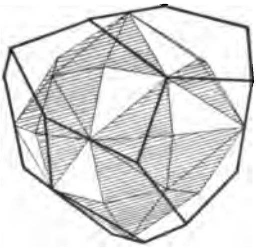

Fig. 37.

Ogdohedrism. There are still other forms, which possess but one-eighth of the number of faces of the original holohedral form, and these are termed *ogdohedral* forms. Figure 270, page 91, shows such ogdohedral forms in combination.

It is evident that these complete and partial forms possess different elements of symmetry and, hence, only those forms which are of the same type, that is, possess the same elements of symmetry, can enter into combination with each other. Compare page 14.

Correlated Forms. Obviously, by the application of hemihedrism, tetartohedrism, and ogdohedrism the original holohedral forms yield *two*, *four*, or *eight* new *correlated* forms, respectively. Most of these correlated forms are *congruent* and differ from each other only in respect to their position in space. A rotation through some definite angle is all that is necessary to have such forms occupy the same position. Such forms are then designated as *plus* and *minus*, or *positive* and *negative*, forms, page 27. Others, however, are related to one another as is the right hand to the left, hence cannot be superimposed and these are said to be *enantiomorphous*. These are designated as *right* and *left* forms. Compare page 37.

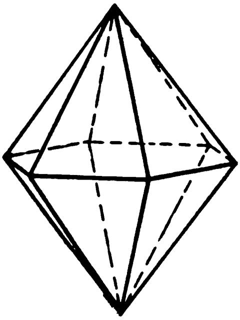

Fig. 38.

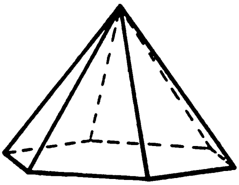

Fig. 39.

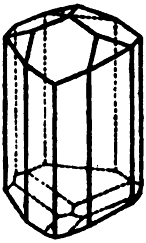

Fig. 40.

Hemimorphism. This is a peculiar type of hemihedrism. It is only possible upon those holohedral and hemihedral forms, or their combinations, which possess a so-called *singular*[1] axis of symmetry. By its application the forms occurring about one end of this axis differ from those about the other. Such forms are *hemimorphic*, and the singular axis of symmetry is said to be *polar*. Hence, the plane of symmetry perpendicular to the singular axis is lost. Compare figures 38, 39, and 40.

---

1) An axis which occurs but once and differs from all others. For example, the axis of tetragonal symmetry in figure 30.

CUBIC SYSTEM.[1]

Crystallographic Axes. All forms which can be referred to three equal and perpendicular axes belong to this system. Figure 41 shows the axial cross. One axis is held vertically, a second extends from front to rear, and the third from right to left. These axes are all interchangeable, each being designated by *a*. Since there are no unknown elements of crystallization in this system (page 9), all substances, regardless of their chemical composition, crystallizing in this system with forms having the same parametral ratios must of necessity possess the same interfacial angles.

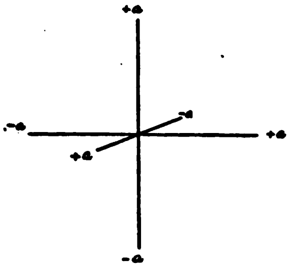

Fig. 41.

Classes of Symmetry. The cubic system includes five groups or classes of symmetry. Beginning with the class of highest symmetry, they are:

1) Hexoctahedral Class (*Holohedrism*)
2) Hextetrahedral Class
3) Dyakisdodecahedral Class } (*Hemihedrism*)
4) Pentagonal icositetrahedral Class
5) Tetrahedral pentagonal dodecahedral Class (*Tetartohedrism*)

Of these classes, the first three are most important, since they possess many representatives among the minerals.

1. HEXOCTAHEDRAL CLASS.[2]

(*Holohedrism.*)

Elements of Symmetry. a) **Planes.** Forms of this class are characterized by nine planes of symmetry. Three of these are parallel to the planes of the crystallographic axes and, hence, perpendicular to each other. They are the *principal* planes of symmetry. They divide space into eight equal parts called *octants*. The six other planes are each parallel to one of the crystallographic axes and bisect the angles between the other two. These are termed the *secondary* or *common* planes of symmetry. By them space is divided into twenty-four equal parts. The nine planes together divide space into forty-eight equal sections. These nine planes are often indicated thus: 3 + 6 = 9. Figures 42 and 43 illustrate the location of the principal and secondary planes, respectively.

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[1] Also termed the regular, isometric, tesseral, or tessular system.

[2] Termed by Dana the *normal group*.

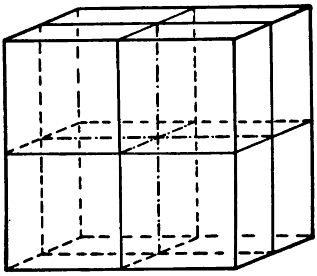
Fig. 42.

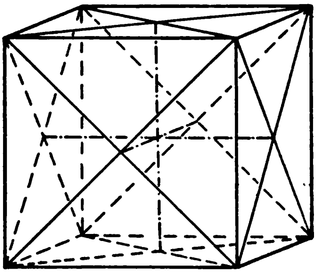
Fig. 43.

b) **Axes.** The intersection lines of the three principal planes of symmetry give rise to the three *principal* axes of symmetry. These are parallel to the crystallographic axes and possess *tetragonal* symmetry, as illustrated by figure 44. The four axes equally inclined to the crystallographic axes are of *trigonal* symmetry, as shown by figure 45. There are also six axes of *binary* symmetry. These lie in the principal planes of symmetry and bisect the angles between the crystallographic axes. Their location is indicated in figure 46. These thirteen axes of symmetry may be indicated as follows:

3■ + 4▲ + 6● = 13.

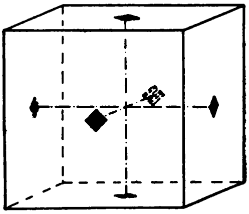
Fig. 44.

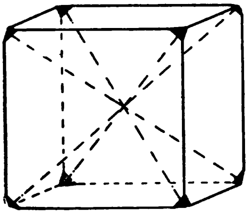
Fig. 45.

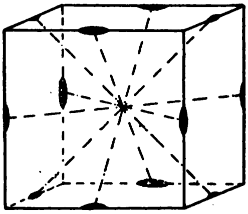
Fig. 46.

c) **Center.** The forms of this class also possess this element of symmetry. Hence, all planes have parallel counter-planes.

The projection of the most general form of this class upon a plane perpendicular to the vertical axis, *i. e.*, in this case a principal plane of symmetry, shows the symmetry relations,1) figure 47.

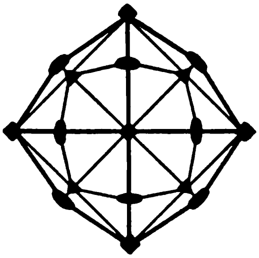

Fig. 47.

FORMS.

1. **Octahedron.** As the name implies, this form consists of eight faces. Each face is equally inclined to the crystallographic axes. Hence, the parametral ratio may be written $(a : a : a)$, which according to Naumann and Miller would be O and $\{1 1 1\}$, respectively. The faces intersect at an angle of 109° 28′ 16″ and in the ideal form, figure 48, are equal, equilateral triangles.

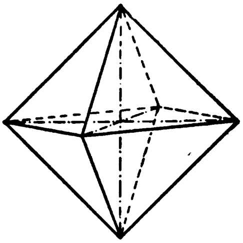

Fig. 48.

The crystallographic axes and, hence, the axes of tetragonal symmetry pass through the tetrahedral angles. The four trigonal axes join the centers of opposite faces, while the six binary axes bisect the twelve edges.

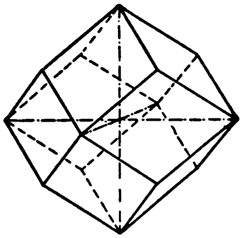

Fig. 49.

2. **Dodecahedron.** This form consists of twelve faces, each cutting two of the crystallographic axes at the same distance, but extending parallel to the third. The symbols are, therefore, $(a : a : \infty a)$, ∞O, $\{110\}$. In the ideal form, figure 49, each face is a rhombus and, hence, the form is often termed the *rhombic dodecahedron*.

The crystallographic axes pass through the tetrahedral angles, the trigonal axes join opposite trihedral angles, and the binary axes the centers of opposite faces. It follows, therefore, that the faces are parallel to the secondary planes of symmetry.

3. **Hexahedron or Cube.** The faces of this form cut one axis and are parallel to the other two. This is expressed by $(a : \infty a$

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1) The heavy lines indicate edges through which principal planes of symmetry pass. The light, full lines show the location of the secondary planes, while dashed lines indicate the absence of planes, see page 26.

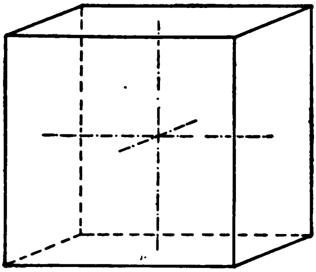
Fig. 50.

: ∞a), $\infty O \infty$, $\{100\}$. Six such faces are possible and when the development is ideal, figure 50, each is a square.

The crystallographic axes pass through the centers of the faces. The trigonal axes of symmetry join opposite trihedral angles, while the binary axes bisect the twelve edges. Compare figures 44, 45, and 46.

4. **Trigonal trisoctahedron.**[1] The faces of this form cut two crystallographic axes at equal distances, the third at a greater distance *ma*. The coefficient *m* is some rational value greater than one but less than infinity. The ratio is $(a : a : ma)$ and it requires twenty-four such faces to enclose space. The Naumann and Miller symbols are mO, $\{hhl\}$, where h is greater than l. Because the general outline of this form is similar to that of the octahedron, each face of which in the ideal forms is replaced by three equal isosceles triangles, it is termed the *trigonal trisoctahedron*, figures 51, $2O\{221\}$, and 52, $3O\{331\}$.

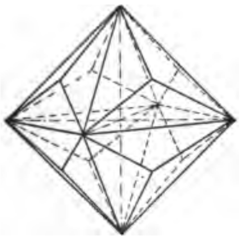
Fig. 51.

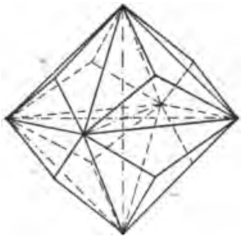
Fig. 52.

The crystallographic axes join opposite octahedral angles. The trigonal axes pass through the trihedral angles and the six binary axes bisect the twelve long edges.

5. **Tetragonal trisoctahedron.**[2] This form consists of twenty-four faces, each cutting one axis at a unit's distance and the other two at greater but equal distances *ma*. The value of *m* is, as above, $m > 1 < \infty$. The symbols are, therefore, $(a : ma : ma)$, mOm, $\{hll\}$, $h > l$. The ideal forms, figures 53, $2O2\{211\}$, and 54, $4O4\{411\}$, bear some resemblance to the octahedron, each face of which has been replaced by three four-sided faces, trapeziums, of

---

[1] Also known as the trisoctahedron.

[2] Also termed the trapezohedron, icositetrahedron, and leucitohedron.

equal size. The form is, therefore, termed the *tetragonal trisoctahedron*. The six tetrahedral angles[1] *a* indicate the position of the crystallographic axes. The trigonal axes of symmetry join opposite trihedral angles, while those of binary symmetry connect the tetrahedral angles[2] *b*.

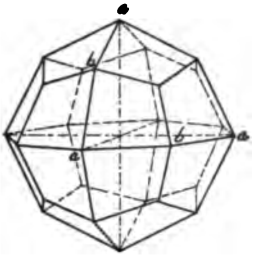

Fig. 53.

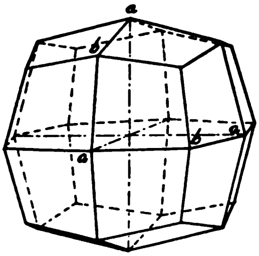

Fig. 54.

6. **Tetrahexahedron.** In this form the faces cut one axis at a unit's distance, the second at the distance *ma*, where $m > 1 < \infty$, and extend parallel to the third axis. The symbols are, therefore, $(a : ma : \infty a)$, $\infty O m$, $\{hk0\}$. The twenty-four faces in the ideal forms, figures 55, $\infty O2\{210\}$, and 56, $\infty O4\{410\}$, are equal isosceles triangles. Since this form may be considered as a cube, whose faces have been replaced by tetragonal pyramids, it is often called the *pyramid cube* or *tetrahexahedron*.

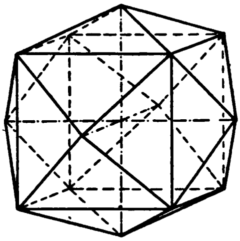
Fig. 55.

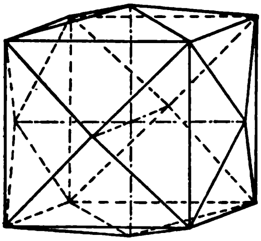
Fig. 56.

The crystallographic axes are located by the six tetrahedral angles. The axes of trigonal symmetry pass through opposite hexahedral angles, while the binary axes bisect the long edges.

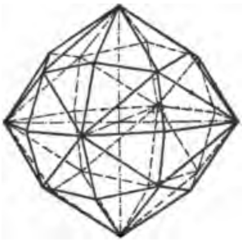
Fig. 57.

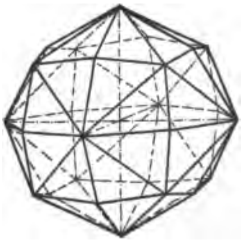
Fig. 58.

7. **Hexoctahedron.** As is indicated by the name, this form is bounded by forty-eight faces. Each cuts one crystallographic axis at a unit's distance, the other two at greater but unequal dis-

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[1] With four equal edges.

[2] These have two pairs of equal edges.

tances na and ma, respectively; n is less than m, the value of m being, as heretofore, $m > 1 < \infty$. Hence, the symbols may be written $(a : na : ma)$, mOn, $\{hkl\}$. Figures 57, $3O\frac{3}{2}\{321\}$, and 58, $5O\frac{5}{3}\{531\}$, show ideal forms, the faces being scalene triangles of the same size.

The crystallographic axes pass through the octahedral angles, while the hexahedral angles locate those of trigonal symmetry. The binary axes pass through opposite tetrahedral angles.

The seven forms just described are the only ones possible in this class. They are often called *simple* forms.

The following table gives a summary of their most important features.

FORMS	SYMBOLS			Number of Equal Faces	Number of Solid Angles			
	Weiss	Naumann	Miller		Trihedral	Tetrahedral	Hexahedral	Octahedral
Octahedron	$a : a : a$	O	$\{111\}$	8	–	6	–	–
Dodecahedron	$a : a : \infty a$	∞O	$\{110\}$	12	8	6	–	–
Hexahedron	$a : \infty a : \infty a$	$\infty O \infty$	$\{100\}$	6	8	—	–	–
Trigonal trisoctahedron	$a : a : ma$	mO	$\{hhl\}$	24	8	—	–	6
Tetragonal trisoctahedron	$a : ma : ma$	mOm	$\{hll\}$	24	8	6[1] + 12[2]	–	–
Tetrahexahedron	$a : ma : \infty a$	$\infty O m$	$\{hk0\}$	24	–	6	8	–
Hexoctahedron	$a : na : ma$	mOn	$\{hkl\}$	48	–	12	8	6

From this tabulation we see that the ratios of the octahedron, dodecahedron, and hexahedron contain no variables and, hence, each is represented by but one form. These are often called *singular* or *fixed* forms. The other ratios, however, contain either one or two variables and, therefore, each represents a series of forms. Compare figures 51 to 58.

---

[1] These have four equal edges.

[2] Two pairs of two equal edges each.

The relationship existing between the simple forms is well expressed by the following diagram:

$$\begin{tikzcd}
 & & a:a:a \arrow[dl, dash] \arrow[dr, dash] & & \\
 & a:a:ma \arrow[dl, dash] \arrow[dr, dash] & & a:ma:ma \arrow[dl, dash] \arrow[dr, dash] & \\
 & & a:na:ma \arrow[d, dash] & & \\
a:a:\infty a \arrow[rr, dash] & & a:ma:\infty a \arrow[rr, dash] & & a:\infty a:\infty a
\end{tikzcd}$$

The three fixed forms are placed at the corners of the triangle and, as is obvious, must be considered as the limiting forms of the others. For example, the value of m in the trigonal trisoctahedron ($a:a:ma$) varies between unity and infinity, page 20. Hence, it follows that the octahedron and dodecahedron are its limiting forms. The tetragonal trisoctahedron ($a:ma:ma$) similarly passes over into the octahedron or cube, depending upon the value of m. The limiting forms are, therefore, in every case readily recognized. Those forms, which are on the sides of the triangle[1], lie in the same zone, that is, their intersection lines are parallel.

Combinations. The following figures illustrate some of the combinations (page 7) of the simple forms, which are observed most frequently.

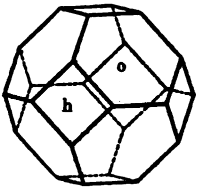

Fig. 59.

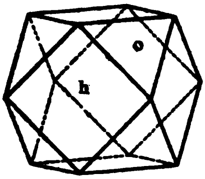

Fig. 60.

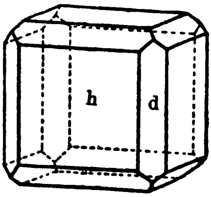

Fig. 61.

Figures 59 and 60. $h = \infty O \infty, \{100\}$; $o = O, \{111\}$. Commonly observed on galena, PbS. In figure 59 the octahedron predominates, whereas in figure 60 both forms are equally developed.

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[1] For example, the octahedron, trigonal trisoctahedron, and dodecahedron.

Figure 61. $h = \infty O \infty, \{100\}$; $d = \infty O, \{110\}$. Copper, Cu, and fluorite, CaF_2, show this combination.

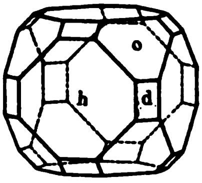

Fig. 62.

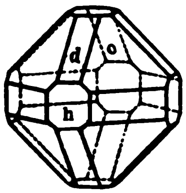

Fig. 63.

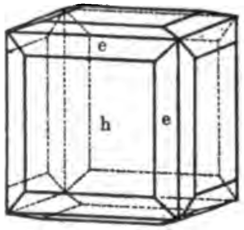

Fig. 64.

Figures 62 and 63. $h = \infty O \infty, \{100\}$; $o = O, \{111\}$; and $d = \infty O, \{110\}$. Also observed on galena, PbS.

Figure 64. $h = \infty O \infty, \{100\}$; $e = \infty O2, \{210\}$. Observed on copper, Cu; fluorite, CaF_2; and halite, NaCl.

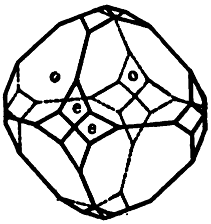

Fig. 65.

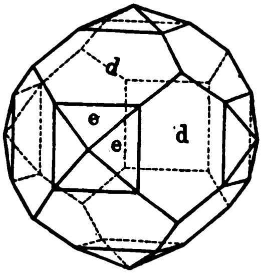

Fig. 66.

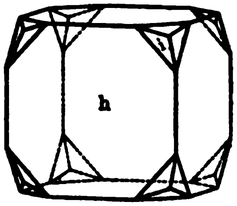

Fig. 67.

Figure 65. $o = O, \{111\}$; $e = \infty O2, \{210\}$. Fluorite, CaF_2.

Figure 66. $d = \infty O, \{110\}$; $e = \infty O2, \{210\}$. Copper, Cu.

Figure 67. $h = \infty O \infty, \{100\}$; $i = 2O2, \{211\}$. Observed on analcite, $NaAl(SiO_3)_2.H_2O$; and argentite, Ag_2S.

Figure 68. $o = O, \{111\}$; $i = 3O3, \{311\}$. Spinel, $MgAl_2O_4$.

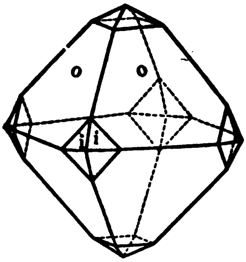

Fig. 68.

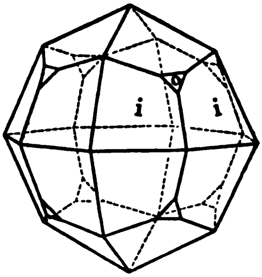

Fig. 69.

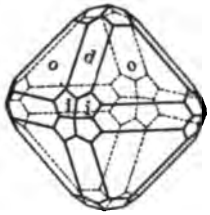

Fig. 70.

Figure 69. $i = 2O2, \{211\}$; $o = O, \{111\}$. Argentite, Ag_2S.

Figure 70. $o = O, \{111\}$; $d = \infty O, \{110\}$; $i = 2O2, \{211\}$. Observed on spinel, $MgAl_2O_4$, and magnetite, Fe_3O_4.

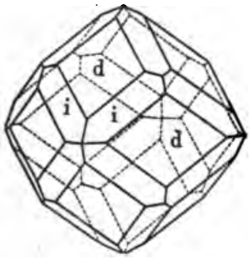

Fig. 71.

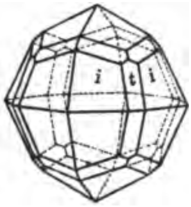

Fig. 72.

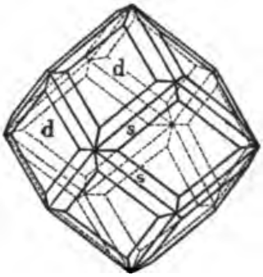

Fig. 73.

Figure 71. $d = \infty O, \{110\}$; $i = 2O2, \{211\}$. A common combination observed on the garnet, $R''_3R'''_2Si_3O_{12}$.

Figure 72. $i = 2O2, \{211\}$; $t = \frac{3}{2}O, \{332\}$. Garnet.

Figure 73. $d = \infty O, \{110\}$; $s = 3O\frac{3}{2}, \{321\}$. Garnet.

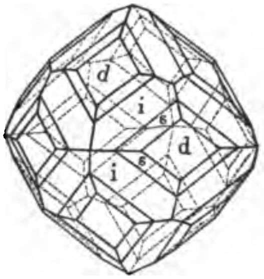

Fig. 74.

Figure 74. $d = \infty O, \{110\}$; $i = 2O2, \{211\}$; $s = 3O\frac{3}{2}, \{321\}$. Garnet.

2. HEXTETRAHEDRAL CLASS.[1]

(*Tetrahedral Hemihedrism.*)

Elements of Symmetry. By decreasing the elements of symmetry of the hexoctahedral class, page 17, the other classes of the cubic system may be deduced. In this class the principal planes of symmetry disappear, which necessitates not only the loss of the six axes of binary symmetry, but also the three of tetragonal symmetry parallel to the crystallographic axes, see page 18. The center of symmetry also disappears. Hence, the elements remaining are: six secondary planes of symmetry, and four trigonal axes, which are now polar. There are also three binary axes parallel to the crystallographic axes, which in the hexoctahedral class are of tetragonal symmetry. The secondary planes of symmetry are easily located since they pass through the edges of the various forms of this class. For the symmetry relations see figure 75.

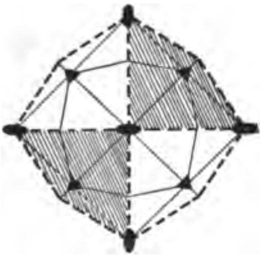

Fig. 75.

Tetrahedral Hemihedrism. The various forms of this class may be derived from the holohedral forms by allowing all faces which occur in alternate octants to be suppressed while the others are extended. This is illustrated by figure 75. A careful study of this figure shows the loss and retention of the elements of symmetry as outlined in the preceding paragraph.

Tetrahedron. By allowing alternate faces of the octahedron to be extended, a new form bounded by four equilateral triangles, intersecting at an angle of 70° 31′ 44″, results. This form is the

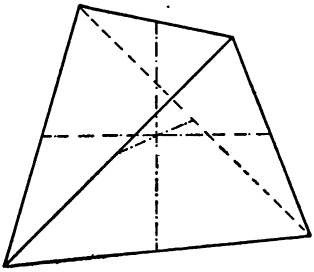

Fig. 76.

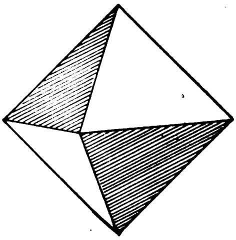

Fig. 77.

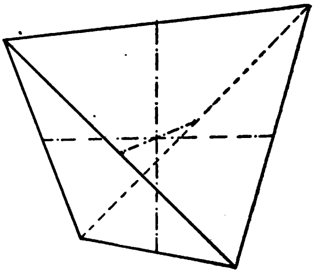

Fig. 78.

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[1] The *tetrahedral group* of Dana.

tetrahedron. Each octahedron, however, yields two new correlated forms as shown by figures 76, 77 and 78. If the shaded faces of figure 77 are suppressed, we obtain the tetrahedron, whose position is indicated by figure 78. However, when the shaded faces are allowed to expand, the others being suppressed, another tetrahedron, figure 76, similar in every respect to the first, but rotated through an angle of 90° is obtained. These forms are, hence, congruent and differentiated by the plus and minus signs, figure 78, *plus* or *positive;* figure 76, *minus* or *negative.* The symbols of the tetrahedron are written:

$$\pm\frac{a:a:a}{2};\ +\frac{O}{2},\kappa\{111\}^{1)};\text{ and }-\frac{O}{2}\kappa\{1\bar{1}1\}.$$

The crystallographic axes pass through the centers of the edges. The axes of binary symmetry are parallel to them; those of trigonal symmetry pass from the trihedral angles to the centers of opposite faces.

Tetragonal tristetrahedron. From the trigonal trisoctahedron, as shown by figures 79, 80, and 81, are obtained two congruent forms bounded by twelve faces, which in the ideal development are similar trapeziums. These forms possess a tetrahedral habit and are called *tetragonal tristetrahedrons*, sometimes also *deltoid dodecahedrons* or simply *deltoids.*

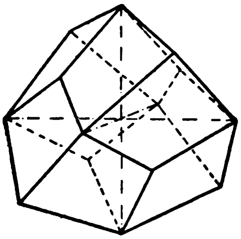
Fig. 79.

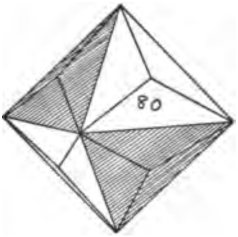

Fig. 80.

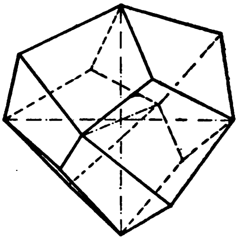
Fig. 81.

The symbols are:

$$\pm\frac{a:a:ma}{2};\quad +\frac{mO}{2},\ \kappa\{h\,h\,l\}\ \text{(figure 81)};\quad -\frac{mO}{2},\ \kappa\{h\,\bar{h}\,l\}$$

(figure 79).

---

1) The letter κ (κλῖνος, inclined) is placed before the Miller indices of forms of this class because their faces are inclined. This type of hemihedrism is often known as the *inclined-face* hemihedrism.

The crystallographic axes pass through opposite tetrahedral angles, while the axes of trigonal symmetry join opposite trihedral angles, one of which is acute, the other obtuse.

Trigonal tristetrahedron. The application of the tetrahedral hemihedrism to the tetragonal trisoctahedron produces two congruent half-forms, bounded by twelve similar isosceles triangles, figures 82, 83, and 84. Since these new forms may be considered as tetrahedrons whose faces have been replaced by trigonal pyramids, they are termed *trigonal tristetrahedrons*[1] or *pyramid tetrahedrons*. Sometimes, moreover, the term *trigonal dodecahedron* is also used.

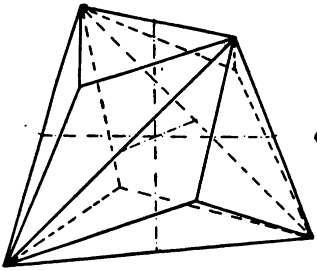

Fig. 82.

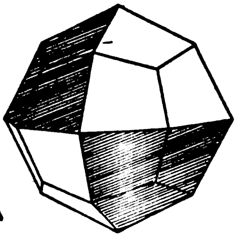

Fig. 83.

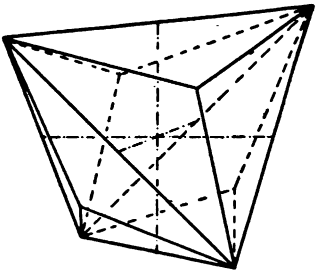

Fig. 84.

The symbols are written:

$$\pm\frac{a:ma:ma}{2};\ +\frac{mOm}{2},\ \kappa\{h\,l\,l\}\ \text{(figure 84)};\ -\frac{mOm}{2},\ \kappa\{h\,\bar{l}\,l\}$$

(figure 82).

The crystallographic axes bisect the long edges. The trigonal axes pass from the trihedral angles to the opposite hexahedral.

Hextetrahedron. In precisely the same manner the hexoctahedron, figure 86, yields two congruent forms of tetrahedral habit, bounded by twenty-four similar scalene triangles. These new forms are called *hextetrahedrons*.

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[1] The following relationship between this and the preceding form and their respective holohedral forms should be carefully noted, viz: The trigonal trisoctahedron (each face is triangular) yields the tetragonal tristetrahedron (faces of tetragonal outline), while, vice versa, the tetragonal trisoctahedron furnishes the trigonal tristetrahedron. Compare figures 79 to 84.

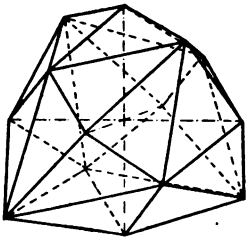

Fig. 85.

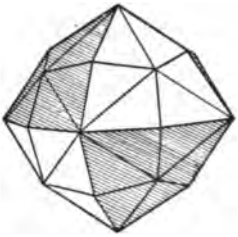

Fig. 86.

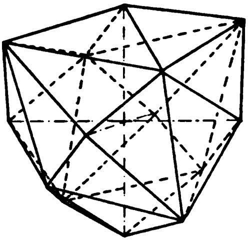

Fig. 87.

The symbols are:

$$\pm\frac{a:na:ma}{2};\ +\frac{mOn}{2},\ \kappa\{hkl\}\ \text{(figure 87)};\ -\frac{mOn}{2},\ \kappa\{h\bar{k}l\}$$

(figure 85.)

The crystallographic axes connect opposite tetrahedral angles. The trigonal axes of symmetry pass through opposite hexahedral angles, one of which is more obtuse than the other.

Hexahedron, dodecahedron, and tetrahexahedron. As is evident from figure 75, no forms, geometrically new, can result from the application of this type of hemihedrism on the cube, dodecahedron, and tetrahexahedron. The faces of these forms belong simultaneously to different octants and, hence, the suppression of a portion of a face in one octant is counterbalanced by a corresponding expansion in another. Therefore, no change in these forms can result. They are from a geometrical standpoint exactly similar to those of the hexoctahedral class. Their symmetry is, however, of a lower grade. This is not to be recognized on models. On crystals, however, the form and position of the so-called *etch figures*, as also the different physical characteristics of the faces, reveal the lower grade of symmetry1). These forms are, therefore, only *apparently holohedral.*

---

1) When crystals are subjected to the action of some solvent for a short time, small depressions or elevations, the so-called *etch figures*, appear. Being dependent upon the internal molecular structure, their form and position indicate the symmetry of the crystal. For instance, figure 88 shows the etch figures on a crystal (cube) of halite, NaCl. Here, it is evident, that the symmetry of the figures in respect to that of the cube is such as to place the crystal in the hexoctahedral class. Figure 89 represents a cube of sylvite, KCl, which geometrically does not differ from the crystal of halite. A lower grade of symmetry is, however, revealed by the position of the etch figures. This crystal belongs to the pentagonal icositetrahedral class, page 36, for no planes of symmetry can be passed through these

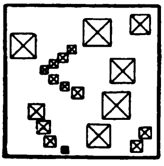

Fig. 88.

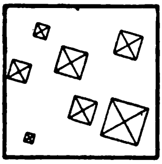

Fig. 89.

The following table shows the important features of the forms of this class of symmetry.

FORMS	SYMBOLS			Number of Equal Faces	Number of Angles		
	Weiss	Naumann	Miller		Trihedral	Tetrahedral	Hexahedral
Tetrahedron	$\pm\frac{a:a:a}{2}$	$+\frac{O}{2}$ $-\frac{O}{2}$	$\kappa\{111\}$ $\kappa\{1\bar{1}1\}$	4	4	—	—
Tetragonal tristetrahedron	$\pm\frac{a:a:ma}{2}$	$+\frac{mO}{2}$ $-\frac{mO}{2}$	$\kappa\{hhl\}$ $\kappa\{h\bar{h}l\}$	12	4+4	6	—
Trigonal tristetrahedron	$\pm\frac{a:ma:ma}{2}$	$+\frac{mOm}{2}$ $-\frac{mOm}{2}$	$\kappa\{hll\}$ $\kappa\{h\bar{l}l\}$	12	4	—	4
Hextetrahedron	$\pm\frac{a:na:ma}{2}$	$+\frac{mOn}{2}$ $-\frac{mOn}{2}$	$\kappa\{hkl\}$ $\kappa\{h\bar{k}l\}$	24	—	6	4 + 4
Dodecahedron	$a:a:\infty a$	∞O	$\kappa\{110\}$	Apparently holohedral			
Tetrahexahedron	$a:ma:\infty a$	∞Om	$\kappa\{hk0\}$				
Hexahedron	$a:\infty a:\infty a$	$\infty O \infty$	$\kappa\{100\}$				

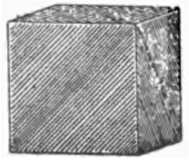

Fig. 90.

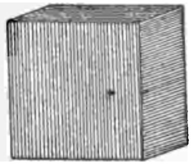

Fig. 91.

figures, which at the same time are planes of symmetry of the cube, as is the case with the crystal of halite, figure 88. For a fuller discussion of these interesting figures consult Groth, Physikalische Krystallographie, 4te Auflage, 1905, 251; also Dana, Text-book of Mineralogy, 1898, 149.

Figures 90 and 91 show two cubes representing crystals of sphalerite, ZnS (figure 90), and pyrite, FeS_2 (figure 91). Although these forms do not differ from the holohedral, it is at once seen

Combinations. Some of the more common combinations are illustrated by the following figures:

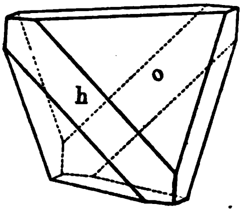

Fig. 92.

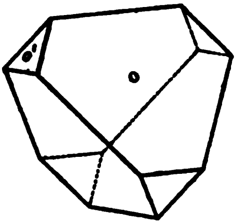

Fig. 93.

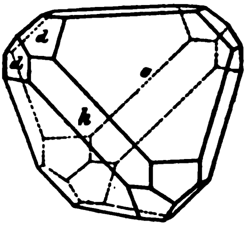

Fig. 94.

Figure 92. $o = +\frac{O}{2}$, $\kappa\{111\}$; $h = \infty O \infty$, $\kappa\{100\}$. Observed on sphalerite, ZnS.

Figure 93. $o = +\frac{O}{2}$, $\kappa\{111\}$; $o' = -\frac{O}{2}$, $\kappa\{1\bar{1}1\}$. Sphalerite.

Figure 94. $o = +\frac{O}{2}$, $\kappa\{111\}$; $h = \infty O \infty$, $\kappa\{100\}$; $d = \infty O$, $\kappa\{110\}$. Tetrahedrite $(Cu_2, Fe, Zn)_4 (As, Sb)_2 S_7$.

Figure 95. $o = +\frac{O}{2}$, $\kappa\{111\}$; $n = +\frac{2O2}{2}$, $\kappa\{211\}$. Tetrahedrite.

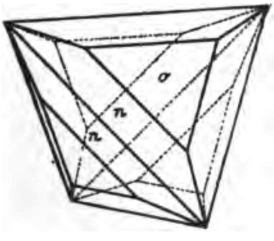

Fig. 95.

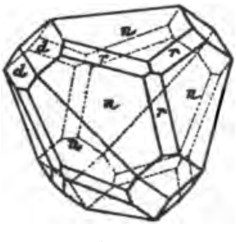

Fig. 96.

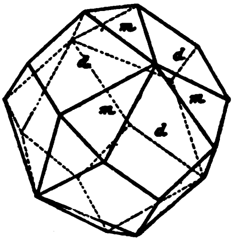

Fig. 97.

Figure 96. $n = +\frac{2O2}{2}$, $\kappa\{211\}$; $r = +\frac{\frac{3}{2}O}{2}$, $\kappa\{332\}$; $d = \infty O$, $\kappa\{110\}$. Tetrahedrite.

---

that adjoining faces possess striations extending in different directions. Figure 90 possesses six secondary planes of symmetry, the principal are wanting, and, hence this cube is only apparently holohedral. It belongs to hextetrahedral class. Figure 91 possesses a still lower grade of symmetry for here only three principal planes are present, and it must, therefore, be referred to the dyakisdodecahedral class, page 32.

Figure 97. $d = \infty O, \kappa\{110\}; \ m = +\frac{3O3}{2}, \kappa\{311\}$. Sphalerite.

Figure 98. $h = \infty O \infty, \kappa\{100\}; \ o = +\frac{O}{2}, \kappa\{111\}; \ o = -\frac{O}{2}, \kappa\{1\bar{1}1\}; \ d = \infty O, \kappa\{110\}; \ n = -\frac{2O2}{2}, \kappa\{2\bar{1}1\}; \ v = +\frac{5O\frac{5}{3}}{2}, \kappa\{531\}$. This combination occurs on boracite, $Mg_7Cl_2B_{16}O_{30}$.

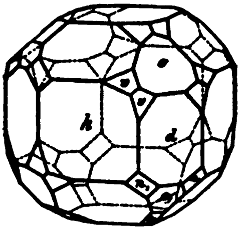

Fig. 98.

3. DYAKISDODECAHEDRAL CLASS.1)

(Pyritohedral Hemihedrism.)

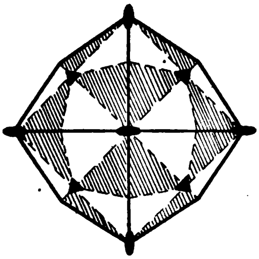

Fig. 99.

Elements of Symmetry. Here the six secondary planes and the six axes of binary symmetry are lost. The elements of symmetry which remain are, therefore, three principal planes parallel to the planes of the crystallographic axes, four trigonal axes, and also the center of symmetry. The three axes parallel to the crystallographic axes now possess binary symmetry. Figure 99 shows these elements as well as the application of the hemihedrism, which will be discussed in the next paragraph.

Pyritohedral hemihedrism 2). In this class the forms may be assumed as derived from the holohedral types by the expansion of the faces lying wholly within alternate spaces formed by the intersection of the secondary planes of symmetry, page 18. Such

---

1) Called by Dana the *pyritohedral group*.

2) Also termed *pentagonal* or *parallel-face* hemihedrism, having reference to the pentagonal outline and parallel arrangement of the faces, respectively.

faces or pairs of faces are then symmetrical to the three principal planes of symmetry, figure 99.

Pyritohedron. The application of this type of hemihedrism to the tetrahexahedron, figure 101, produces two correlated, congruent, hemihedral forms, figures 100 and 102, bounded by twelve similar faces. Each face is an unequilateral pentagon, four sides of which are equal. Since these forms are congruent, a rotation through 90° is all that is necessary to bring them into precisely the same position in space. They are, hence, designated as *plus* and *minus* forms.[1]

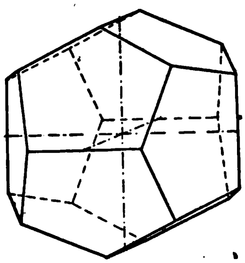

Fig. 100.

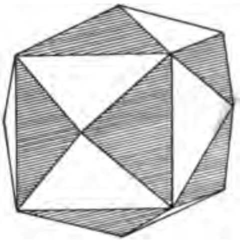

Fig. 101.

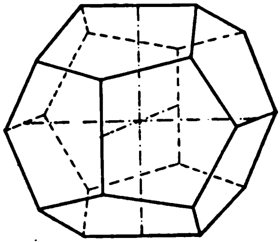

Fig. 102.

The symbols are:

$$\pm\left[\frac{a:ma:\infty a}{2}\right];\ +\left[\frac{\infty O m}{2}\right],\ \pi\{h\,k\,o\}\quad \text{(figure 102);}$$

$$-\left[\frac{\infty O m}{2}\right]\ \pi\{k\,h\,o\}^{2)}\quad \text{(figure 100).}$$

Because this form occurs frequently on pyrite it is termed the *pyritohedron*, although *pentagonal dodecahedron*[3] is quite often used.

The axes of binary symmetry, hence, also the crystallographic axes bisect the six long edges. The trigonal axes pass through the trihedral angles, the edges of which are of equal lengths.

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[1] Also designated as *right* and *left*. It is preferable, however, to use those terms in reference to enantiomorphous forms only, page 16.

[2] In this class, π (παράλληλος, parallel-faced) is placed before the Miller indices, see page 27. To distinguish the other symbols from those of the preceding class, they are enclosed in brackets.

[3] The regular pentagonal dodecahedron of geometry, bounded by equilateral pentagons intersecting in equal edges and angles is crystallographically an impossible form, the value of m being $\frac{1+\sqrt{5}}{2}$, which is irrational, compare page 10.

Dyakisdodecahedron. By the expansion and suppression of alternate pairs of faces crossed by the principal planes of symmetry, the hexoctahedron, figure 104, yields two correlated, congruent forms, each bounded by twenty-four similar trapeziums, figures 103 and 105. These forms are the *dyakisdodecahedrons*, also termed *didodecahedrons*, or *diploids*.

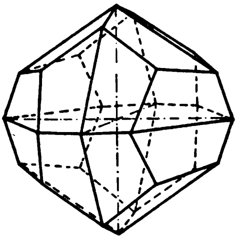

Fig. 103.

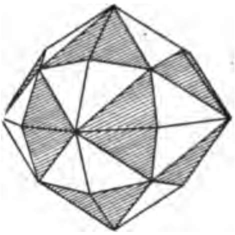

Fig. 104.

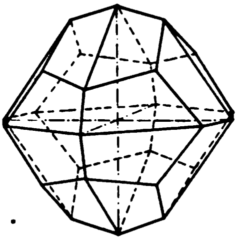

Fig. 105.

The symbols are:

$$\pm\left[\frac{a : na : ma}{2}\right]; +\left[\frac{mOn}{2}\right], \ \pi\{h\,k\,l\} \text{ (figure 105)}:-\left[\frac{mOn}{2}\right],$$

$\pi\{hlk\}$ (figure 103).

The crystallographic axes, also those of binary symmetry, pass through the six tetrahedral angles possessing two pairs of equal edges. The trigonal axes join opposite trihedral angles.

Other Forms. The hexahedron, octahedron, dodecahedron, trigonal trisoctahedron, and tetragonal trisoctahedron, the other forms of the holohedral type, do not yield new forms by the application of this hemihedrism. This is clearly shown by figure 99, illustrating the elements of symmetry of this class. Being apparently holohedral, these forms may occur independently or in combination, but always with the full number of faces. Their symmetry is, however, less than that of the holohedrons. See page 29.

In the following table the important features of the forms of this class are indicated.

FORMS	SYMBOLS			Number of Faces	Number of Angles			
	Weiss	Naumann	Miller		Trihedral		Tetrahedral	
					3 Equal Edges	2 + 1 Edges	2 + 2 Edges	2 + 1 + 1 Edges
Octahedron	$a : a : a$	O	$\pi\{111\}$	Apparently holohedral				
Dodecahedron	$a : a : \infty a$	∞O	$\pi\{110\}$					
Hexahedron	$a : \infty a : \infty a$	∞O∞	$\pi\{100\}$					
Trigonal trisoctahedron	$a : a : ma$	mO	$\pi\{hhl\}$					
Tetragonal trisoctahedron	$a : ma : ma$	mOm	$\pi\{hll\}$					
Pyritohedron	$\pm\left[\frac{a:ma:\infty a}{2}\right]$	$+\left[\frac{\infty Om}{2}\right]$ $-\left[\frac{\infty Om}{2}\right]$	$\pi\{hko\}$ $\pi\{kho\}$	12	8	12	—	—
Dyakisdodecahedron	$\pm\left[\frac{a:na:ma}{2}\right]$	$+\left[\frac{mOn}{2}\right]$ $-\left[\frac{mOn}{2}\right]$	$\pi\{hkl\}$ $\pi\{hlk\}$	24	8	–	6	12

Combinations. The accompanying figures show some combinations of the forms of this class.

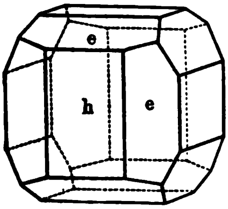

Fig. 106.

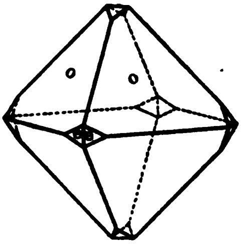

Fig. 107.

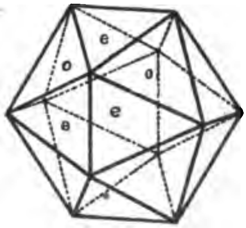

Fig. 108.

Figure 106. $h = \infty O \infty$, $\pi\{100\}$; $e = +\left[\frac{\infty O2}{2}\right]$, $\pi\{210\}$. Commonly observed on pyrite, FeS_2.

Figures 107 and 108. $o = O$, $\pi\{111\}$; $e = +\left[\frac{\infty O2}{2}\right]$, $\pi\{210\}$.

In figure 107 the octahedron predominates, whereas in figure 108 both forms are about equally developed. Pyrite.

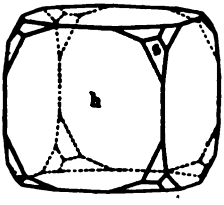

Fig. 109.

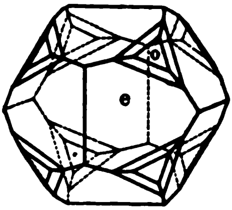

Fig. 110.

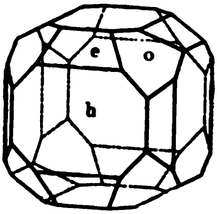

Fig. 111.

Figure 109. $h = \infty O \infty$, $\pi\{100\}$; $s = +\left[\frac{3O\frac{3}{2}}{2}\right]$, $\pi\{321\}$. Pyrite.

Figure 110. $e = +\left[\frac{\infty O2}{2}\right]$, $\pi\{210\}$; $o = O$, $\pi\{111\}$. Between o and e but in the same zone, $+\left[\frac{3O\frac{3}{2}}{2}\right]\pi\{321\}$. Pyrite.

Figure 111. $h = \infty O \infty$, $\pi\{100\}$; $o = O$, $\pi\{111\}$; $e = +\left[\frac{\infty O2}{2}\right]\pi\{210\}$. Cobaltite, CoAsS.

4. PENTAGONAL ICOSITETRAHEDRAL CLASS.[1]

(Plagihedral or Gyroidal Hemihedrism.)

Elements of Symmetry. All planes, as well as the center of symmetry, are lost. There remain, however, all of the thirteen axes, namely, three tetragonal, four trigonal, and six binary axes. Figure 112 shows these elements and also the method of the application of the plagihedral hemihedrism.

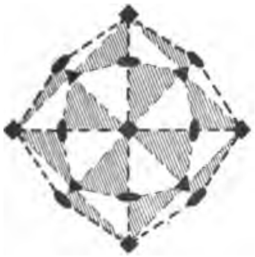

Fig. 112.

Plagihedral Hemihedrism. Forms of this class may be assumed as derived from the holohedrons by the alternate expansion and suppression of faces lying wholly within each of the forty-eight sections, resulting from the intersection of the nine planes of symmetry, page 18. Compare figure 112.

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[1] Termed by Dana the *plagihedral group*.

Pentagonal Icositetrahedron. By the extension and suppression, respectively, of alternate faces of the hexoctahedron, figure 114, two new correlated forms, bounded by twenty-four equal unsymmetrical pentagons result. These forms are termed *pentagonal icositetrahedrons* or *gyroids*. They are not congruent, for they cannot be brought into the same position by rotation and, hence, are not superimposable. Therefore, they are *enantiomorphous*, page 16, and distinguished as *right*, figure 115, and *left*, figure 113, according to which of the top faces (right or left) in the front upper octant has been extended.

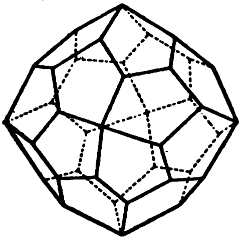
Fig. 113.

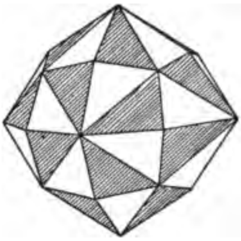
Fig. 114.

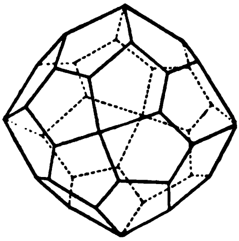
Fig. 115.

The symbols are:

$$r.\ l\,\frac{a:na:ma}{2};\ r\,\frac{mOn}{2},\ \gamma\{khl\}^{1)};\ l\,\frac{mOn}{2},\ \gamma\{hkl\}.$$

The crystallographic axes, also those of tetragonal symmetry, join opposite tetrahedral angles. The trigonal axes pass through those trihedral angles which possess equal edges, while the six binary axes bisect the twelve edges intermediate between those forming the tetrahedral angles.

Other forms. The six other holohedrons are not from a geometrical standpoint affected by this hemihedrism. A study of figure 112 shows that no new forms can result, for on these holohedrons none of the faces lie wholly within one of the forty-eight sections referred to on page 36. The octahedron, dodecahedron, hexahedron, the trigonal and tetragonal trisoctahedrons, and the tetrahexahedron are only apparently holohedral.

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1) In this class, γ γυρός, bent) is placed before the Miller indices. Compare footnotes on pages 27 and 33.

The principal features of the forms of this class may be tabulated as follows:

FORMS	SYMBOLS			Number of Faces	Number of Angles		
	Weiss	Naumann	Miller		Trihedral 3 Equal Edges	Trihedral 3 Unequal Edges	Tetrahedral
Octahedron	$a : a : a$	O	$\gamma\{111\}$				
Dodecahedron	$a : a : \infty a$	∞O	$\gamma\{110\}$				
Hexahedron	$a : \infty : \infty a$	$\infty O \infty$	$\gamma\{100\}$				
Trigonal trisoctahedron	$a : a : ma$	mO	$\gamma\{hhl\}$		Apparently holohedral		
Tetragonal trisoctahedron	$a : ma : ma$	mOm	$\gamma\{hll\}$				
Tetrahexahedron	$a : ma : \infty a$	∞Om	$\gamma\{hko\}$				
Pentagonal Icositetrahedron	$r, l\,\frac{a : na : ma}{2}$	$r\frac{mOn}{2}$ $l\frac{mOn}{2}$	$\gamma\{khl\}$ $\gamma\{hkl\}$	24	8	24	6

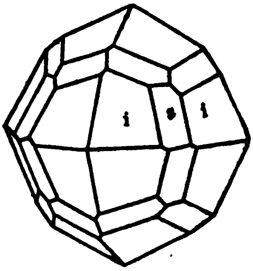

Fig. 116.

Combinations. Figure 116. $i = 2O2$, $\gamma\{211\}$; $s = l\,\frac{\frac{8}{5}O\frac{8}{7}}{2}$, $\gamma\{875\}$. Sal Ammoniac, NH_4Cl.

5. TETRAHEDRAL PENTAGONAL DODECAHEDRAL CLASS.[1]

(*Tetartohedrism.*)

Elements of Symmetry. The nine planes, the three axes of tetragonal and the six of binary symmetry, as also the center, are lost. The elements of this class are, hence, four polar axes of trigonal and three of binary symmetry, the latter being parallel to the crystallographic axes. See figure 117.

---

[1] The *tetartohedral group* of Dana.

Tetartohedrism. The simple tetartohedral forms may be conceived as derived from ~~the hemihedrons of~~ any ~~one~~ of the three hemihedral classes—tetrahedral, pyritohedral, or plagihedral—by the ~~subsequent~~ application ~~of one~~ of the ~~other~~ two methods. The result is in each case the same. Compare figures 118, 119, 120, 122, and 125.

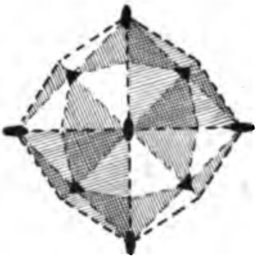
Fig. 117.

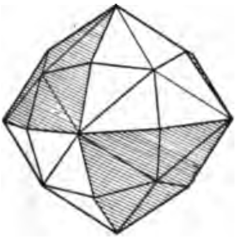
Fig. 118.

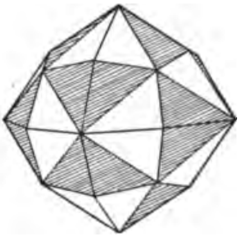
Fig. 119.

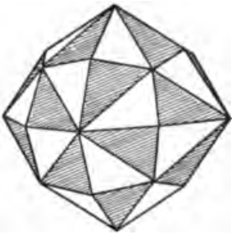
Fig. 120.

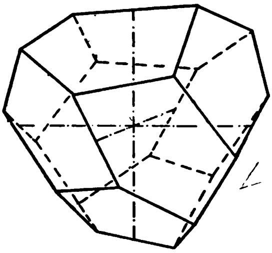
Fig. 121.

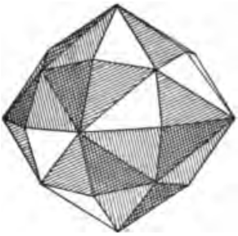
Fig. 122.

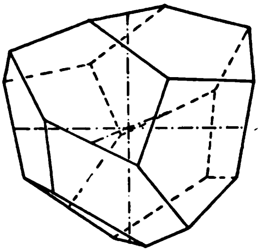
Fig. 123.

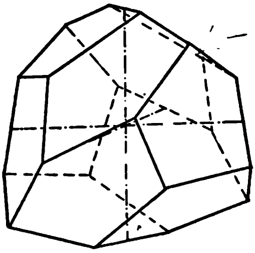
Fig. 124.

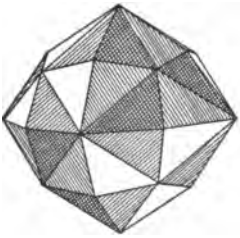
Fig. 125.

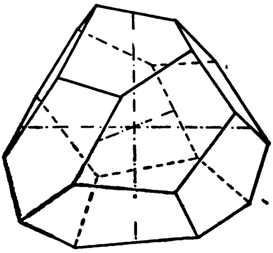
Fig. 126.

Tetrahedral pentagonal dodecahedron. From the hexoctahedron by the ~~successive~~ application of two types of hemihedrism four new forms of tetrahedral habit, bounded by twelve unsymmetrical pentagons, result. The derivation of these forms is easily to be seen from figures 121 - 126, also figure 37, page 15. In figure 122, the form derived by the expansion of the unshaded faces is known as the *left positive tetrahedral pentagonal dodecahedron*, whereas the one derived from the shaded faces alternating with these — *i. e.*, in the same octant — is the *right positive* form. From figure 125 the *left negative* and *right negative* forms are likewise possible.

The symbols according to Weiss are $\pm\ r, \pm\ l\ \frac{a : na : ma}{4}$. According to Naumann and Miller they may be written:

1. $+r\frac{mOn}{4}$, $\kappa\pi\{khl\}$[1], *Positive right*, figure 123.
2. $-r\frac{mOn}{4}$, $\kappa\pi\{h\bar{k}l\}$, *Negative right*, figure 126.
3. $+l\frac{mOn}{4}$, $\kappa\pi\{hkl\}$, *Positive left*, figure 121.
4. $-l\frac{mOn}{4}$, $\kappa\pi\{k\bar{h}l\}$, *Negative left*, figure 124.

Forms 1 and 2, also 3 and 4, are among themselves congruent, whereas the pairs 1 and 3, and 2 and 4, are enantiomorphous.

The crystallographic axes bisect the six equal edges, while the four trigonal axes join opposite trihedral angles of which one is more obtuse than the other. These trihedral angles possess equal edges.

Other forms. If tetartohedrism be applied to the other holohedrons, it follows that four of them will yield forms, as follows:

Octahedron — two tetrahedrons, figures 76 and 78, page 26.

Trigonal trisoctahedron — two tetragonal tristetrahedrons, figures 79 and 81, page 27.

Tetragonal trisoctahedron — two trigonal tristetrahedrons, figures 82 and 84, page 28.

Tetrahexahedron — two pyritohedrons, figures 100 and 102, page 33.

The first three forms do not differ geometrically from those obtained by the tetrahedral hemihedrism, while the fourth is the same as that derived by the pyritohedral hemihedrism. They are, hence, only apparently hemihedral forms. The cube and dodecahedron are unchanged geometrically and, therefore, apparently holohedral.

---

[1] Two letters, each representing one of the types of hemihedrism applied, are used with the Miller indices in this class.

The following table shows the principal features of the forms of this class:

FORMS	SYMBOLS			Number of Faces	Number of Angles	
					Trihedral	
	Weiss	Naumann	Miller		Equal Edges	Unequal Edges
Dodecahedron	$a:a:\infty a$	∞O	$\kappa\pi\{110\}$	Apparently holohedral		
Hexahedron	$a:\infty a:\infty a$	$\infty O \infty$	$\kappa\pi\{100\}$			
Tetrahedrons	$\pm\frac{a:a:a}{2}$	$+\frac{O}{2}$ $-\frac{O}{2}$	$\kappa\pi\{111\}$ $\kappa\pi\{1\bar{1}1\}$	Apparently tetrahedral hemihedrons		
Tetragonal tristetrahedrons	$\pm\frac{a:a:ma}{2}$	$+\frac{mO}{2}$ $-\frac{mO}{2}$	$\kappa\pi\{hhl\}$ $\kappa\pi\{h\bar{h}l\}$			
Trigonal tristetrahedrons	$\pm\frac{a:ma:ma}{2}$	$+\frac{mOm}{2}$ $-\frac{mOm}{2}$	$\kappa\pi\{hll\}$ $\kappa\pi\{h\bar{l}l\}$			
Pyritohedrons	$\pm\left[\frac{a:ma:\infty a}{2}\right]$	$+\left[\frac{\infty Om}{2}\right]$ $-\left[\frac{\infty Om}{2}\right]$	$\kappa\pi\{hk0\}$ $\kappa\pi\{kh0\}$	Apparently pyritohedral hemihedrons		
Tetrahedral pentagonal dodecahedrons	$\frac{a:na:ma}{4}$ $\pm r, \pm l$	$+r\frac{mOn}{4}$ $-r\frac{mOn}{4}$ $+l\frac{mOn}{4}$ $-l\frac{mOn}{4}$	$\kappa\pi\{khl\}$ $\kappa\pi\{h\bar{k}l\}$ $\kappa\pi\{hkl\}$ $\kappa\pi\{k\bar{h}l\}$	12	4+4[1]	12

---

[1] One set is more obtuse than the other.

Combinations. In this class it is evident that the apparently holohedral and hemihedral forms of the various classes, as described on pages 40 and 41, may occur together on the same crystal. For example, figures 127 and 128 show crystals of sodium chlorate, $NaClO_3$, where $h = \infty O \infty$, $\kappa\pi\{100\}$; $d = \infty O$, $\kappa\pi\{110\}$; $-o = -\frac{O}{2}$, $\kappa\pi\{1\bar{1}1\}$; p (figure 127) $= -\left[\frac{\infty O2}{2}\right]$, $\kappa\pi\{120\}$; p (figure 128) $= +\left[\frac{\infty O2}{4}\right]$, $\kappa\pi\{210\}$.

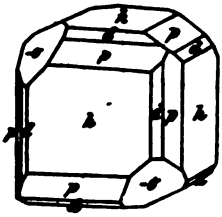

Fig. 127.

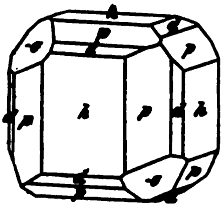

Fig. 128.

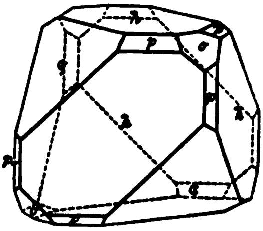

Fig. 129.

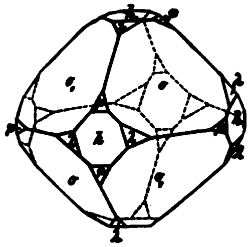

Fig. 130.

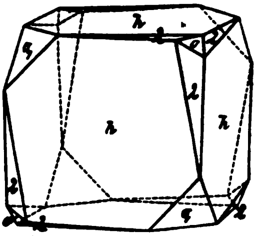

Fig. 131.

Figures 129, 130, and 131 illustrate crystals of barium nitrate, $Ba(NO_3)_2$, with the following forms: $h = \infty O \infty$, $\kappa\pi\{100\}$; $o = +\frac{O}{2}$, $\kappa\pi\{111\}$; $o_1 = -\frac{O}{2}$, $\kappa\pi\{1\bar{1}1\}$; $p = -\left[\frac{\infty O2}{2}\right]$, $\kappa\pi\{120\}$; $\lambda = +\frac{4O2}{2}l$, $\kappa\pi\{421\}$.

HEXAGONAL SYSTEM.

Crystallographic Axes. This system includes all forms which can be referred to four axes, three of which are equal and lie in a horizontal plane, and intersect each other at an angle of 60°. These are termed the *secondary* or *lateral* axes, being designated by the letter *a*. These axes are interchangeable. The fourth, a *principal* axis, is perpendicular to the plane of the secondary axes and is termed the *c* axis. It may be longer or shorter than the secondary axes. The three equal axes, which bisect the angles between the secondary axes, are the *intermediate* axes. These may be designated by *b*. Figure 132 shows an axial cross of this system.

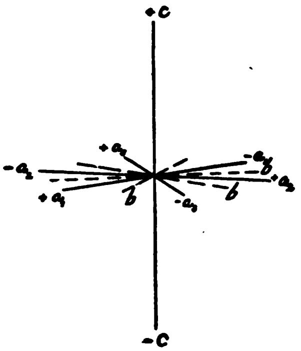

Fig. 132.

In reading crystals of the hexagonal system, it is customary to hold the *c* axis vertical, letting one of the secondary or *a* axes extend from right to left. The extremities of the secondary axes are alternately characterized as plus and minus, see figure 132. In referring a form to the crystallographic axes, it is common practice to consider them in the following order: a_1 first, then a_2, thirdly a_3, and lastly the *c* axis. The symbols always refer to them in this order. It is also to be noted that in following this order, one of the lateral axis will always be preceded by a minus sign.

Since the lengths of the *a* and *c* axes differ, it is necessary to assume for each substance crystallizing in this system a fundamental form, whose intercepts are taken as representing the unit lengths of the secondary and principal axes, respectively. The ratio, which exists between the lengths of these axes is called the *axial* ratio and is always an irrational value, the *a* axis being assumed as unity, page 7.

Classes of Symmetry. The hexagonal system includes a larger number of classes of symmetry than any other system, namely, twelve. The order in which they will be discussed is as follows:

*	1.	Dihexagonal bipyramidal class	(*Holohedrism*).
*	2.	Dihexagonal pyramidal class	*Holohedrism and hemimorphism.*
‡ †	3.	Ditrigonal bipyramidal class	(*Hemihedrism*).
‡ *	4.	Ditrigonal scalenohedral class	
*	5.	Hexagonal bipyramidal class	
	6.	Hexagonal trapezohedral class	
‡ *	7.	Ditrigonal pyramidal class	*Hemihedrism and hemimorphism.*
	8.	Hexagonal pyramidal class	
‡ †	9.	Trigonal bipyramidal class	(*Tetartohedrism.*)
‡ *	10.	Trigonal trapezohedral class	
‡ *	11.	Trigonal rhombohedral class	
‡	12.	Trigonal pyramidal class	*Tetartohedrism and hemimorphism* (*Ogdohedrism*).

Those classes marked with an * are the most important, for nearly all of the crystals of this system belong to some one of them. No representatives have as yet been observed for the classes marked by †. Those marked ‡ are often grouped together and form the *trigonal* system.

1. DIHEXAGONAL BIPYRAMIDAL CLASS.[1]

(*Holohedrism.*)

Symmetry. This class possesses the highest grade of symmetry of any in the hexagonal system.

a) **Planes.** In all there are *seven* planes of symmetry. One of these, the *principal* plane, is parallel to the plane of the secondary axes, hence, horizontal. The other planes are divided into two series of three each, which are termed the *secondary* and *intermediate*, respectively. Each of the secondary planes includes the *c* or principal axis and one of the secondary, *a*, axes. These planes are, therefore, vertical and perpendicular to the principal plane.

---

[1] The *normal group* of Dana.

They intersect at angles of 60°. The intermediate planes are also vertical and perpendicular to the principal planes, for they bisect the angles between the secondary planes and, hence, each includes the c and one of the intermediate axes.

The secondary and principal planes divide space into twelve equal parts, called *dodecants;* the seven planes, however, into twenty-four parts, figure 133.

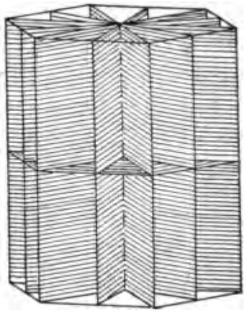
Fig. 133.

These planes are often designated as follows:

1 Principal + 3 Secondary + 3 Intermediate = 7 Planes.

b) **Axes.** Parallel to the vertical or c axis is an axis of *hexagonal* symmetry, while the axes parallel to the secondary and intermediate axes possess *binary* symmetry. These axes are often indicated, thus,

1 ⬢ + 3 ⬬ + 3 ⬬ = 7 axes.

c) **Center.** This element of symmetry is also present, requiring every face to have a parallel counter-face. Figure 134, the projection of the most complicated form upon a plane perpendicular to the vertical axis, shows the elements of symmetry of this class.

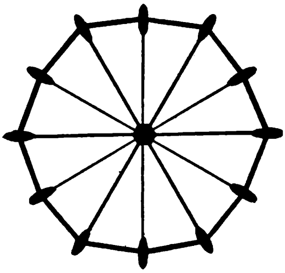
Fig. 134.

1. **Hexagonal bipyramid of the first order.** From figure 132, it is obvious that any plane which cuts any two adjacent secondary axes at the unit distance from the center must extend parallel to the third. If such a plane be assumed to cut the c axis at its unit length from the center, the parametral ratio would then be

$$a_1 : \infty a_2 : a_3 : c.$$

According to the above elements of symmetry, twelve planes possessing this ratio are possible. They enclose space and give rise to the form termed the *hexagonal bipyramid*[1] *of the first order,*

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[1] **Since these are really double pyramids, the term *bipyramid* is employed.**

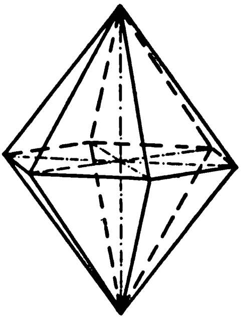

Fig. 135.

figure 135. In the ideal form, the faces are all equal, isosceles triangles. The symbols are $(a\colon \infty a\colon a\colon c)$, P, $\{10\bar{1}1\}$.[1] Because the intercepts along the c and two secondary axes are taken as units, such bipyramids are also known as *fundamental* or *unit* bipyramids, page 6.

Planes are, however, possible which cut the two secondary axes at the unit distances, but intercept the c axis at the distance mc, the coefficient m being some rational value smaller or greater than 1, see page 6. Such bipyramids, according as m is greater or less than unity, are more acute or obtuse than the fundamental form. They are termed *modified* hexagonal bipyramids of the first order. Their symbols are $(a\colon \infty a\colon a\colon mc)$, mP, $\{h\,o\,\bar{h}\,l\}$, where $m = \frac{h}{l}$, also $m > 0 < \infty$.

The principal axis passes through the hexahedral angles, the secondary axes join tetrahedral angles, while the intermediate bisect the horizontal edges. Hence when such bipyramids are held correctly, a face is directed towards the observer. The various axes of symmetry are located by means of the above.

2. **Hexagonal bipyramid of the second order.** In form, this bipyramid does not differ from the preceding. It is, however,

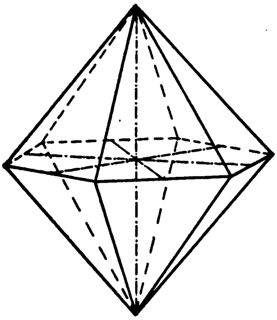

Fig. 136.

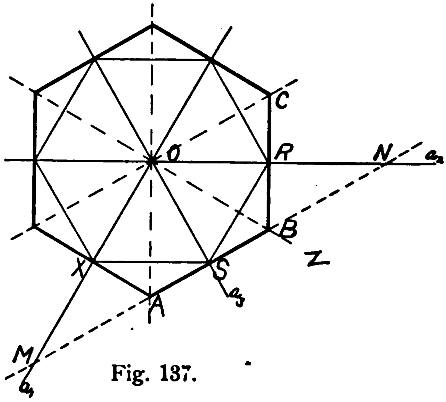

Fig. 137.

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[1] In this system it is advantageous to employ the indices as modified by Bravais $(h\,i\,\bar{k}\,l)$ rather than those of Miller, who uses but three.

to be distinguished by its position in respect to the secondary axes. The bipyramid of the second order is so held that an edge, and not a face, is directed towards the observer. This means that the secondary axes are perpendicular to and bisect the horizontal edges as shown in figure 136. Figure 137 shows the cross section including the secondary axes. From these figures it is obvious that each face cuts one of the secondary axes at a unit distance, the other two at greater but equal distances. For example, AB cuts a_3 at the unit distance OS, and a_1 and a_2 at greater but equal distances OM and ON, respectively.

The following considerations will determine the length of OM and ON, the intercepts on a_1 and a_2, in terms of OS $= 1$.

As already indicated, the secondary axes are perpendicular to the horizontal edges, hence OS and ON are perpendicular to AB and BC, respectively. Therefore, in the right triangles ORB and NRB, the side RB is common and the angles OBR and NBR are equal.[1]

Therefore, OR $=$ RN. But OR $=$ OS $= 1$.

Hence, ON $=$ OR $+$ RN $= 2$.

In the same manner it can be shown that the intercept on a_1 is equal to that along a_2, that is, twice the unit length. The parametral ratio of the hexagonal bipyramid of the second order, therefore, is $(2a : 2a : a : mc)$, or expressed according to Naumann and Bravais, mP2 and $\{h\,h\,\overline{2h}\,l\}$, where $\frac{2h}{l} = m$. Figure 137 shows the positions of the bipyramids of both orders in respect to the secondary axes, the inner outline representing that of first, the outer the one of the second order.

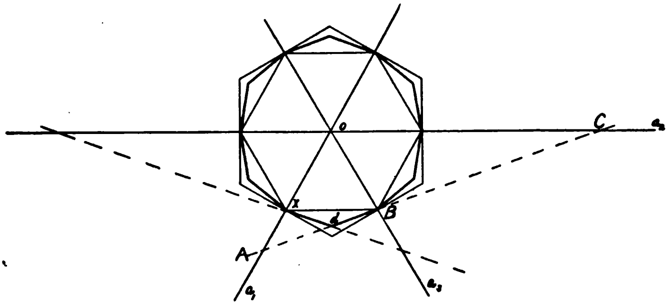

Fig. 138.

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[1] For, angle ABC equals 120°, angle NBR is then 60°, being the supplement of ABC. But the intermediate axis OZ bisects the angle ABC, hence angle OBR is also 60°.

3. **Dihexagonal bipyramid.** The faces of this form cut the three secondary axes at unequal distances. For example, in figure 138 the face represented by dB cuts the a_1 axis at A, a_2 at C, and a_3 at B. Assuming the shortest of these intercepts as unity, hence, OB = a = 1, we at once see that one of these axes is cut at a unit's distance from O, the other two, however, at greater distances. If we let the intercepts OA and OC be represented by $n(\mathrm{OB}) = na$, and $p(\mathrm{OB}) = pa$, respectively, the ratio will read

$$na : pa : a : mc, \quad m\mathrm{P}n, \quad \{hi\bar{k}l\}.$$

In this ratio $p = \frac{n^{1)}}{n-1}$. Twenty-four planes having this ratio are possible and give rise to the form called the *dihexagonal bipyramid*, figure 139. In the ideal form the faces are equal, scalene triangles, cutting in twenty-four polar[2], a and b, and twelve equal basal[3] edges. The polar edges and angles are alternately dissimilar. This is shown by figure 140, where the heavy inner outline represents the form of the first order, the outer the one of the second, and the intermediate outlines the dihexagonal type in respect to the secondary axes.

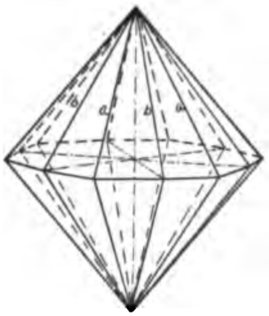

Fig. 139.

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[1] From the above discussion, it follows that in figure 138

$$\mathrm{OA} : \mathrm{OC} : \mathrm{OB} = n : p : 1.$$

Draw XB parallel to a_2, and then

$$\mathrm{OA} : \mathrm{XA} = \mathrm{OC} : \mathrm{XB}.$$

But the triangle OXB is equilateral,

Hence, $\mathrm{XB} = \mathrm{XO} = \mathrm{OB} = 1.$
And $\mathrm{XA} = \mathrm{OA} - \mathrm{OX} = \mathrm{OA} - 1.$
Therefore, $\mathrm{OA} : \mathrm{OA} - 1 = \mathrm{OC} : 1.$
But $\mathrm{OA} = n$, and $\mathrm{OC} = p$,
Hence, $n : n - 1 = p : 1,$
Or $\frac{n}{n-1} = p.$

It can also be shown that the algebraic sum of the Miller-Bravais indices $\{hi\bar{k}\}$ is equal to zero, *i. e.*,

$$h + i + \bar{k} = o.$$

For, $\mathrm{OA} = \frac{1}{k}$, $\mathrm{OC} = \frac{1}{i}$, and $\mathrm{OB} = \frac{1}{k}$, page 12.

Therefore, $\frac{1}{h} : \frac{1}{h} - \frac{1}{k} = \frac{1}{i} : \frac{1}{k}$,

Or, $k = h + i.$

Hence, $h + i + \bar{k} = o.$

[2] Those joining the horizontal and principal axes.

[3] These lie in the principal plane of symmetry.

These three hexagonal bipyramids are closely related, for, if we suppose the plane represented by AB, figure 140, to be rotated about the point B so that the intercept along a_2 increases in length, the one

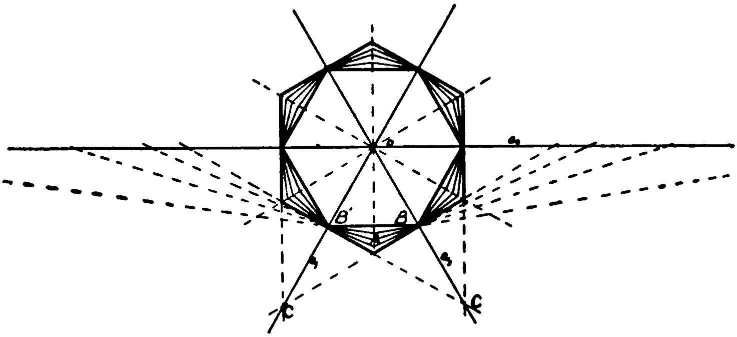


Fig. 140.

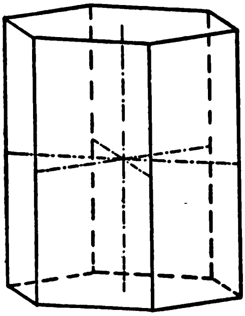
along a_1 decreases until it equals oB$'$ = oB = 1. Then the plane is parallel to a_3 and the ratio for the bipyramid of the first order results. If, however, AB is rotated so that the intercept along a_2 is decreased in length, the one along a_1 increases until it equals oC = 2oB$'$ = $2a$. When this is the case, the intercept on a_2 is also equal to $2a$, for then the plane is perpendicular to a_3. This gives rise to the ratio of the bipyramid of the second order.

That the bipyramids of the first and second orders are the limiting forms of the dihexagonal bipyramid is also shown by the fact that $p = \frac{n}{n-1}$. For, if $n = 1$, it follows that $p = \infty$, hence, the ratio of the form of the first order. But, when $n = 2$, $p = 2$ also, therefore, the ratio for the second order results. With dihexagonal bipyramids the following holds good:

$$n > 1 < 2, \text{ and } p > 2 < \infty.$$

The dihexagonal bipyramid whose polar edges and angles are all equal is crystallographically not a possible form, because the value of n would then be $\frac{1}{2}(1 + \sqrt{3}) = \sqrt{2}.\ \sin 75^\circ = 1.36603+$, which of course is irrational. It also follows that in those dihexagonal bipyramids, where the value of n is less than $1.36603+$, for instance, $\frac{6}{5} = 1.20$, the more acute pole angles indicate the location of the secondary axes, the more obtuse that of the intermediate, and *vice* *versa*, when n is greater than 1.36603+, for example, $\frac{8}{5} = 1.60$. This is clearly shown by figure 140.

Hexagonal prism of the first order. This form is easily derived from the bipyramid of the same order by allowing the intercept along the c axis to assume its maximum value, infinity. Then the twelve planes of the bipyramid are reduced to six, each plane cutting two secondary axes at the unit distance and extending parallel to the c axis. The symbols are $(a : \infty a : a : \infty c)$, ∞P, $\{10\bar{1}0\}$. This form cannot enclose space and, hence, may be termed an *open form*, page 6. It cannot occur independently and is always to be observed in combination, figure 141. The secondary axes join opposite edges, *i. e.*, a face is directed towards the observer when properly held.



Fig, 141.

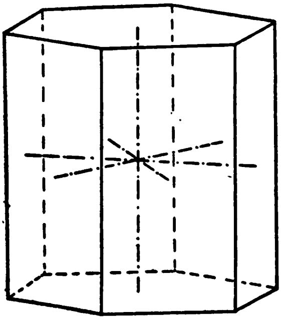


Fig. 142.

Hexagonal prism of the second order. This prism bears the same relation to the preceding form that the bipyramid of the second order does to the one of the first, page 46. The symbols are $(2a : 2a : a : \infty c)$, $\infty P2$, $\{11\bar{2}0\}$. It is, hence, an open form consisting of six faces. The secondary axes join the centers of opposite faces, hence, an edge is directed towards the observer, figure 142.

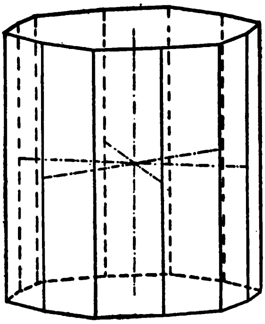


Fig. 143.

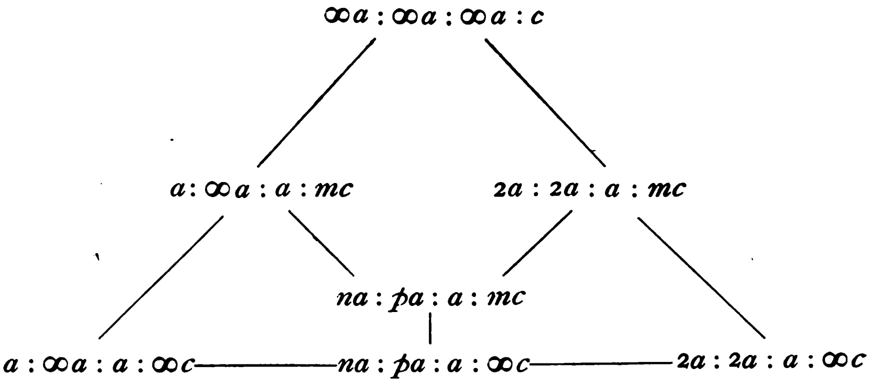
Dihexagonal prism. This form may be obtained from the corresponding bipyramid by increasing the value of m to infinity, which gives $(na : pa : a : \infty c)$, ∞Pn, $\{hi\bar{k}o\}$. This prism consist of twelve faces whose alternate intersection angles are dissimilar. This form, figure 143, is closely related to the corresponding bipyramid and, hence, all that has been said concerning the dihexagonal bipyramid, page 48, in respect to the location of the secondary axes and its limiting forms might be repeated here, substituting, of course, for the bipyramids of the first and second orders the corresponding prisms.

7. **Hexagonal basal pinacoid.** The faces of this form are parallel to the principal plane of symmetry and possess the following symbols $(\infty a : \infty a : \infty a : c)$, OP, $\{0001\}$. It is evident from the presence of a center and principal plane of symmetry that two such planes are possible. This, like the prisms, is an open form and must always occur in combination. Figure 141 shows this form in combination with the prism of the first order.

These are the seven simple forms possible in this system. Their principal features may be summarized as follows:

FORMS	SYMBOLS			Number of Faces	Solid Angles		
	Weiss	Naumann	Miller-Bravais		Tetrahedral	Hexahedral	Dodecahedral
Unit Bipyramid First order	$a : \infty a : a : c$	P	$\{10\bar{1}1\}$	12	6	2	—
Modified Bipyramids First order	$a : \infty a : a : mc$	mP	$\{h0\bar{h}l\}$	12	6	2	—
Bipyramids Second order	$2a : 2a : a : mc$	$mP2$	$\{hh\overline{2h}l\}$	12	6	2	—
Dihexagonal Bipyramids	$na : pa : a : mc$	mPn	$\{hi\bar{k}l\}$	24	6+6	—	2
Prism First order	$a : \infty a : a : \infty c$	∞P	$\{10\bar{1}0\}$	6	—	—	—
Prism Second order	$2a : 2a : a : \infty c$	$\infty P2$	$\{11\bar{2}0\}$	6	—	—	—
Dihexagonal Prisms	$na : pa : a : \infty c$	∞Pn	$\{hi\bar{k}0\}$	12	—	—	—
Basal Pinacoid	$\infty a : \infty a : \infty a : c$	OP	$\{0001\}$	2	—	—	—

Relation of forms. The following diagram, similar to the one for the cubic system, page 23, expresses very clearly the relationship existing between the various simple forms.



Combinations. The following figures illustrate some of the combinations of forms of this class.

Figure 144, p = P, $\{10\bar{1}1\}$; o = mP, $\{h\,o\,\bar{h}\,l\}$.

Figure 145, p = P, $\{10\bar{1}1\}$; n = P2 $\{11\bar{2}1\}$.

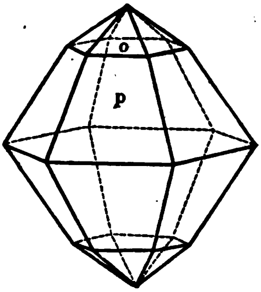


Fig. 144.

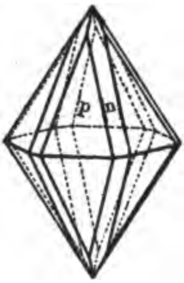


Fig. 145.

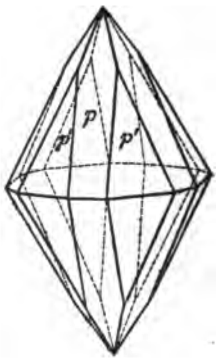


Fig. 146.

Figure 146, p = P, $\{10\bar{1}1\}$; p' = $\frac{4}{3}$P 2, $\{22\bar{4}3\}$.

Figure 147, $m = \infty$P, $\{10\bar{1}0\}$; p = P, $\{10\bar{1}1\}$.

Figure 148, $m = \infty P$, $\{10\bar{1}0\}$; $p = P$, $\{10\bar{1}1\}$; $u = 2P$, $\{20\bar{2}1\}$; $c = OP$, $\{0001\}$; $s = 2P2$; $\{1121\}$; and $v = 3P\frac{3}{2}$, $\{21\bar{3}1\}$. This combination occurs on beryl, $Be_8Al_2(SiO_8)_6$.

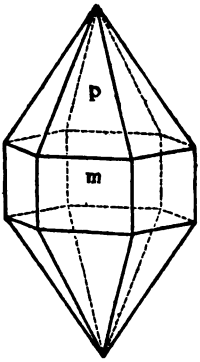


Fig 147.

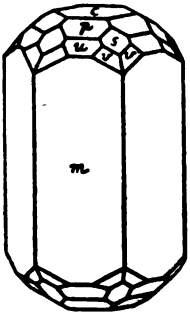


Fig. 148.

2. *DIHEXAGONAL PYRAMIDAL CLASS.*[1]

(*Holohedrism with Hemimorphism.*)

Symmetry. The forms of this class are hemimorphic, that is, the principal plane of symmetry disappears and the singular, c, axis is now polar, page 16. The disappearance of the principal plane necessitates also the loss of the center and six binary axes of symmetry. Hence, the remaining elements are three secondary and three intermediate planes, and one polar axis of hexagonal symmetry parallel to the c axis. Figure 149 shows the elements of symmetry of this class.[2]

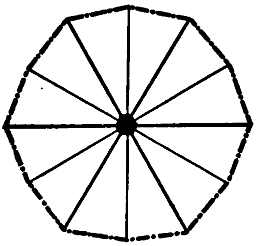


Fig. 149.

Forms. The forms of this class differ from those of the preceding (holohedral) in that the faces occurring about either pole[3] are to be considered as belonging to independent forms. Hence, each of the three bipyramids yields two new forms, called *upper* and *lower* pyramids, respectively. For example, the dihexa-

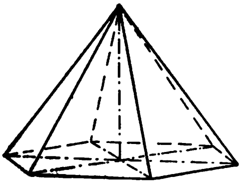


Fig. 150.

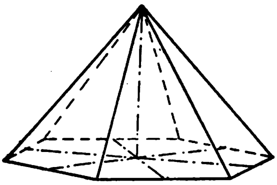


Fig. 151.

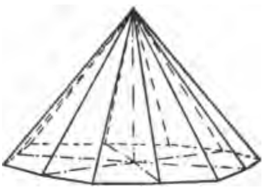


Fig. 152.

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[1] *Hemimorphic group* of Dana.

[2] The heavy dotted and dashed line not only indicates the absence of the principal plane but also shows that hemimorphism is effective. Compare pages 26, 32, and 36.

[3] The ends of the principal or c axis are often spoken of as *poles*.

gonal bipyramid yields the *upper* and *lower* dihexagonal pyramids,[1] each possessing twelve faces. Figures 150, 151, and 152 show the upper forms of the three bipyramids. These pyramids are open forms. They are shown in combination with the lower basal pinacoid, for it is also divided into an *upper* and *lower* form of one face each. The symbols of these forms are the same as those used for the holohedral, except that the end of the c axis is indicated about which they occur. Since, however, the development of the prisms is the same about both poles, no new forms result from them.

The principal features and symbols of the forms of this class are given in the following table:

FORMS	SYMBOLS			Number of Faces	Solid Angles		Number of Edges
	Weiss	Naumann	Miller-Bravais		Hexahedral	Dodecahedral	
Upper and Lower Pyramids First Order	$\frac{a:\infty a:a:mc}{2}\ u,\ l$	$\frac{mP}{2}u$ $\frac{mP}{2}l$	$\{h0\bar{h}l\}$ $\{h0\bar{h}\bar{l}\}$	6	1	—	6
Upper and Lower Pyramids Second Order	$\frac{2a:2a:a:mc}{2}\ u,\ l$	$\frac{mP2}{2}u$ $\frac{mP2}{2}l$	$\{hh\overline{2h}l\}$ $\{hh\overline{2h}\bar{l}\}$	6	1	—	6
Upper and Lower Dihexagonal Pyramids	$\frac{na:pa:a:mc}{2}\ u,\ l$	$\frac{mPn}{2}u$ $\frac{mPn}{2}l$	$\{hi\bar{k}l\}$ $\{hi\bar{k}\bar{l}\}$	12	—	1	6+6
Prism First Order	$a:\infty a:a:\infty c$	∞P	$\{10\bar{1}0\}$	6	Same as in holohedral class		
Prism Second Order	$2a:2a:a:\infty c$	$\infty P2$	$\{11\bar{2}0\}$	6			
Dihexagonal Prism	$na:pa:a:\infty c$	mPn	$\{hi\bar{k}0\}$	12			
Upper and Lower Basal Pinacoids	$\frac{\infty a:\infty a:\infty a:c}{2}\ u,\ l$	$\frac{OP}{2.}u,\ l$	$\{0001\}$ $\{000\bar{1}\}$	1	—	—	—

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1) The term *pyramid* in itself suggests hemimorphism, since it is not a doubly terminated form as is the *bipyramid*.

Combinations.

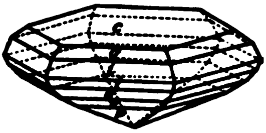

Fig. 153.

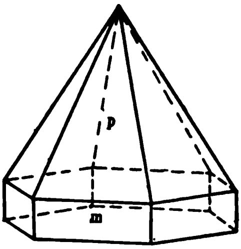

Fig. 154.

Figure 153, $c = \frac{OP}{2} u$, $\{0001\}$; $o = \frac{P}{2} u \;\&\; l$, $\{10\bar{1}1\}$ & $\{10\bar{1}\bar{1}\}$; $i = \frac{2P}{2} u \;\&\; l$, $\{20\bar{2}1\}$ & $\{20\bar{2}\bar{1}\}$; $v = \frac{\frac{2}{3}P}{2} l$, $\{20\bar{2}3\}$, and $\mu = \frac{\frac{1}{2}P}{2} l$, $\{10\bar{1}2\}$; observed on Iodyrite, AgI.

Figure 154, $p = \frac{P}{2} u$, $\{10\bar{1}1\}$; $m = \infty P$, $\{10\bar{1}0\}$, observed on Zincite, ZnO.

HEXAGONAL HEMIHEDRISMS.

In the hexagonal system four types of hemihedrism, as illustrated by figures 155 to 158, are possible.

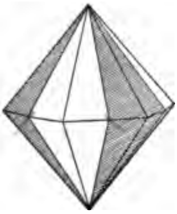

Fig. 155.

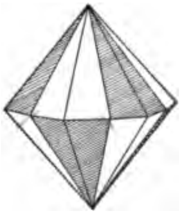

Fig. 156.

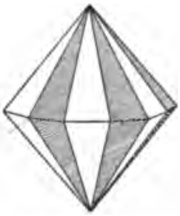

Fig. 157.

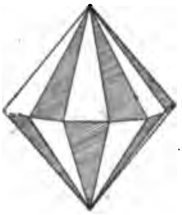

Fig. 158.

a) **Trigonal hemihedrism.** The three secondary planes of symmetry divide space into six equal sections.[1] All faces in alternate sections are extended, the others suppressed, figure 155.

b) **Rhombohedral hemihedrism.** The three secondary planes together with the principal plane divide space into twelve sections, called dodecants, page 45. Faces in alternate dodecants are subject to extension, figure 156.

c) **Pyramidal hemihedrism.** By means of the three secondary and three intermediate planes twelve sections, figure 157, result. Faces in alternate sections are suppressed.

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[1] Liebisch, however, refers to the sections derived by the intersection of the intermediate instead of the secondary planes. The forms, which result, are in both cases identical. Their positions in respect to the secondary axes differ by an angle of 30°. Compare Liebisch, Grundriss der Physikalischen Krystallographie, 1896, 112.

d) **Trapezohedral hemihedrism.** All planes of symmetry possible in this system divide space into twenty-four sections. The extension of faces occurs in alternate sections of this character, figure 158.

Of these hemihedrisms, the rhombohedral and pyramidal types are the most important. No representative of the trigonal hemihedrism has yet been observed.

3. DITRIGONAL BIPYRAMIDAL CLASS.

(*Trigonal Hemihedrism.*)

Symmetry. From figure 155 showing this method of hemihedrism it is obvious that the secondary planes of symmetry are lost. With them the center and the axis of hexagonal symmetry also disappear. The elements of this class are, hence, one principal and three intermediate planes, three binary axes parallel to the intermediate, and an axis of trigonal symmetry parallel to the *c* axis. The binary axes are polar. Figure 159 not only shows these elements but the application of the hemihedrism as well.

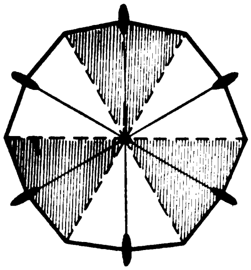

Fig. 159.

Trigonal bipyramids. From the hexagonal bipyramid of the first order two correlated, congruent forms, each bounded by six equal isosceles triangles, result. These forms and their position in respect to the secondary axes are shown by figures 160-165. They are the trigonal bipyramids. The symbols are $\pm\left[\frac{a:\infty a:a:mc}{2}\right]$; $\pm\frac{mP}{2}\{ho\bar{h}l\}$, figure 162; $-\frac{mP}{2}$, $\{oh\bar{h}l\}$, figure 160.

The axis joining the trihedral angles is of one of trigonal symmetry, those passing through the centers of the horizontal edges to the opposite tetrahedral angles are of binary symmetry. These are parallel to the intermediate axes.

Trigonal prisms. The corresponding hexagonal prism yields two trigonal prisms,[1] each bounded by three faces, as is shown by figures 163 to 168.

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[1] **Being open forms, they are shown in combination with the basal pinacoid.**

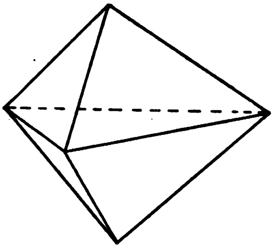
Fig. 160.

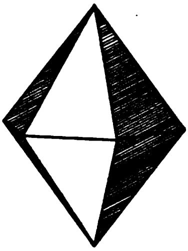
Fig. 161.

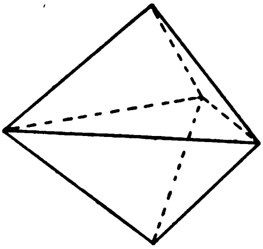
Fig. 162.

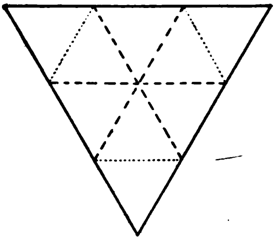
Fig. 163.

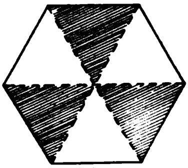
Fig. 164.

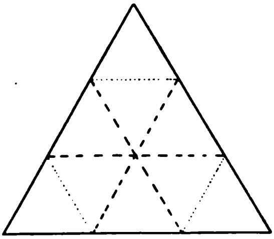
Fig. 165.

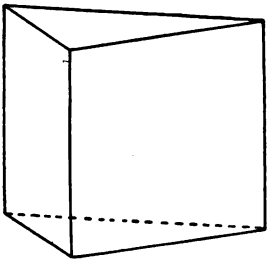
Fig. 166.

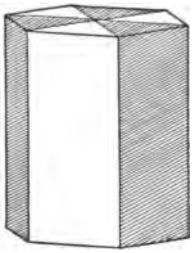
Fig. 167.

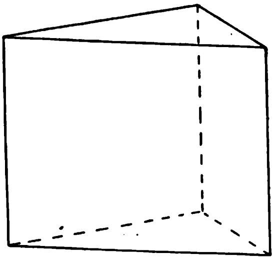
Fig. 168.

The symbols are: $\pm\left[\frac{a:\infty a:a:\infty c}{2}\right]$; $+\frac{\infty P}{2}$, $\{h o \bar{h} o\}$, figure 168; $-\frac{\infty P}{2}$, $\{o h \bar{h} o\}$, figure 166.

The trigonal axis is parallel to the intersection lines of the prism faces, while those of binary symmetry pass from the centers of the faces to those of the opposite edges, figures 166 and 168.

Ditrigonal bipyramids. Every dihexagonal bipyramid, when subjected to the trigonal hemihedrism, gives rise to two correlated and congruent forms, bounded by twelve scalene triangles, which are known as the *ditrigonal bipyramids*. The derivation and position of these forms are illustrated by figures 169 to 174.

The symbols are: $\pm \left\{\frac{na : pa : a : mc}{2}\right\}$, $+\frac{m\mathrm{P}n}{2}$, $\{hi\bar{k}l\}$, figure 171; $-\frac{m\mathrm{P}n}{2}$, $\{ih\bar{k}l\}$, figure 169.

The trigonal axis joins the hexahedral angles, those of binary symmetry extend from an obtuse to an opposite more acute tetrahedral angle.

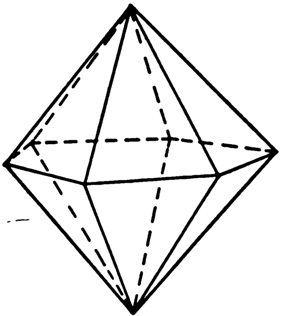
Fig. 169.

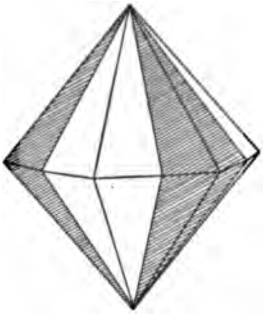
Fig. 170.

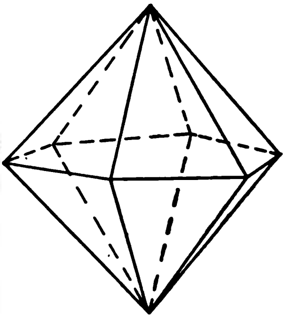
Fig. 171.

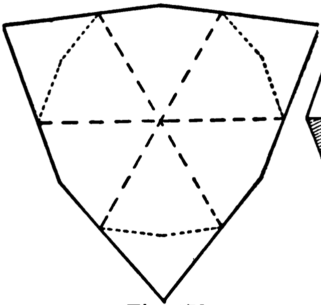
Fig. 172.

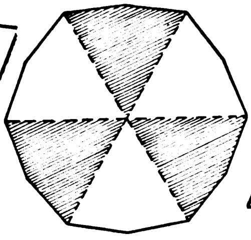
Fig. 173.

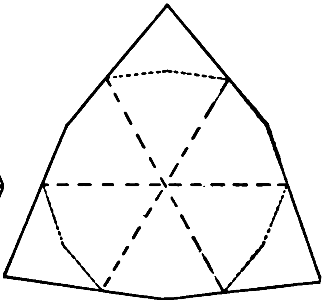
Fig. 174.

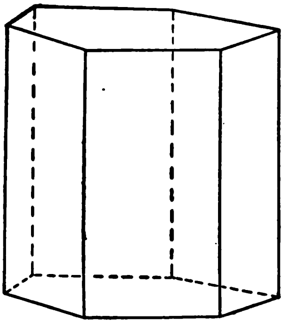
Fig. 175.

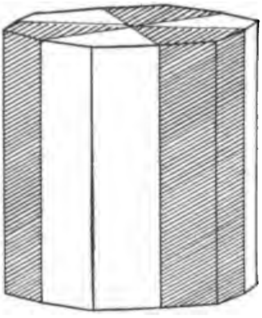
Fig. 176.

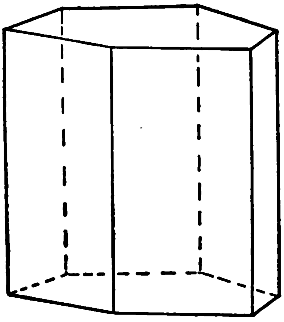
Fig. 177.

Ditrigonal prisms. The dihexagonal prism yields two ditrigonal prisms, as shown by figures 172 to 177. The symbols are $\pm\left[\frac{na:pa:a:\infty c}{2}\right]$, $+\frac{\infty \mathrm{P}n}{2}$, $\{h i \bar{k} o\}$, figure 177; $-\frac{m\mathrm{P}n}{2}$, $\{i h \bar{k} o\}$ figure 175.

The trigonal axis is parallel to the intersection lines of the prism faces, those of binary symmetry are parallel to the intermediate axes.

Other forms. The bipyramids and prism of the second order as also the basal pinacoid are not changed geometrically by the trigonal hemihedrism, for none of their faces lie wholly within the sections formed by the secondary axes. Compare figures 159 and 164. These forms are, therefore, apparently holohedral.

No representative of this class has yet been discovered.

The chief characteristic of the forms may be tabulated as follows:

FORMS	SYMBOLS			Number of Faces	Solid Angles		
	Weiss	Naumann	Miller-Bravais		Trigonal	Tetrahedral	Hexahedral
Trigonal Bipyramids First order	$\pm\left[\frac{a:\infty a:a:mc}{2}\right]$	$+\frac{mP}{2}$ $-\frac{mP}{2}$	$\{h o \bar{n} k\}$ $\{o h \bar{h} k\}$	6	2	3	—
Hexagonal Bipyramids Second order	$2a:2a:a:mc$	$mP2$	$\{h h \overline{2h} l\}$	Apparently holohedral			
Ditrigonal Bipyramids	$\pm\left[\frac{na:pa:a:mc}{2}\right]$	$+\frac{mPn}{2}$ $-\frac{mPn}{2}$	$\{h i \bar{k} l\}$ $\{i h \bar{k} l\}$	12	—	3+3	2
Trigonal Prisms First order	$\pm\left[\frac{a:\infty a:a:\infty c}{2}\right]$	$+\frac{\infty P}{2}$ $-\frac{\infty P}{2}$	$\{h o \bar{h} o\}$ $\{o h \bar{h} o\}$	3	—	—	—
Hexagonal Prism Second order	$2a:2a:a:\infty c$	$\infty P2$	$\{1120\}$	Apparently holohedral			
Ditrigonal Prisms	$\pm\left[\frac{na:pa:\infty a:c}{2}\right]$	$+\frac{\infty Pn}{2}$ $-\frac{\infty Pn}{2}$	$\{h i \bar{k} o\}$ $\{i h \bar{k} o\}$	6	—	—	—
Basal Pinacoid	$\infty a:\infty a:\infty a:c$	OP	$\{0001\}$	Apparently holohedral			

4. DITRIGONAL SCALENOHEDRAL CLASS.1)

(*Rhombohedral Hemihedrism.*)

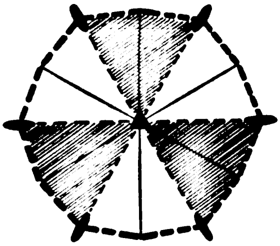

Fig. 178.

Symmetry. The principal and secondary planes of symmetry together with the hexagonal axis are lost. The following elements are present: three intermediate planes, three axes of binary and one of trigonal symmetry, also the center. Figure 178 shows these elements and the application of the rhombohedral hemihedrism, page 55.

Rhombohedrons. From the hexagonal bipyramid of the first order two new congruent forms are the result of this type of hemihedrism, figures 181 to 183. In the ideal development each of these forms is bounded by six equal rhombs and are called *positive* and *negative rhombohedrons.* The lateral edges form a zigzag line about the form. The six polar edges form two equal trihedral angles, bounded by equal edges. These may be larger or smaller than the other trihedral angles, according to the value of $a : c$.2)

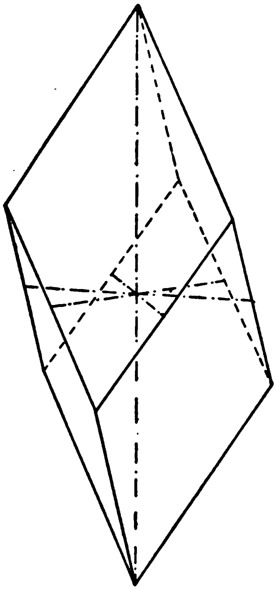

Fig. 179.

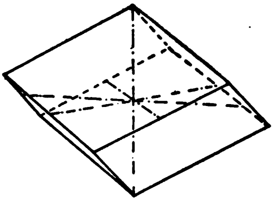

Fig. 180.

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1) Dana terms this class the *rhombohedral group*.

2) The cube, when held so that one of its axes of trigonal symmetry, page 18, is vertical, may be considered as a rhombohedron whose edges and angles are equal. The ratio, $a:c$, in this case would be $1 : \sqrt{1.5} = 1 : 1.2247+$. Those rhombohedrons, therefore, whose c axes have a greater value than $1.2247+$ have pole angles less than 90°. When, however, the value is less than $1.2247+$, the pole angles are then greater than 90° and, hence, such rhombohedrons may be spoken of as *acute* and *obtuse*, respectively, figures 179 and 180.

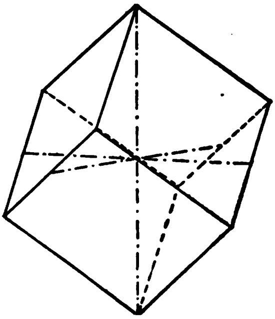
Fig. 181.

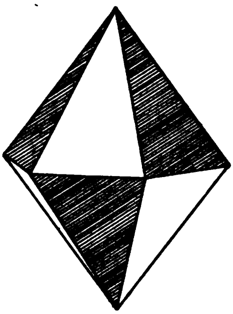
Fig. 182.

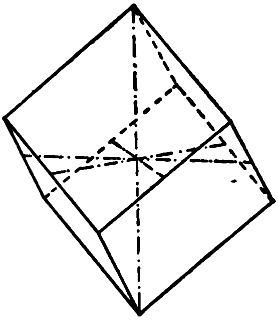
Fig. 183.

The symbols are $\pm \left[\frac{a : \infty a : a : mc}{2}\right]$; $+\frac{mP}{2}$, $\kappa\{h o \bar{h} l\}$, figure 183; $-\frac{mP}{2}$, $\kappa\{o h \bar{h} l\}$, figure 181.

The principal crystallographic axis passes through the two equal trihedral angles, the secondary axes bisect opposite lateral edges. These axes indicate the directions of those of trigonal and binary symmetry, respectively.

Scalenohedrons. Every dihexagonal bipyramid gives rise to two congruent forms, bounded by twelve similar scalene triangles, called *scalenohedrons*, of which one is *positive* and the other *negative*, figures 184 to 186. Each possesses six obtuse and six more acute polar edges, also six zigzag lateral edges. As is the case with the rhombohedrons, obtuse and acute scalenohedrons are possible, depending upon the value of $a : c$.

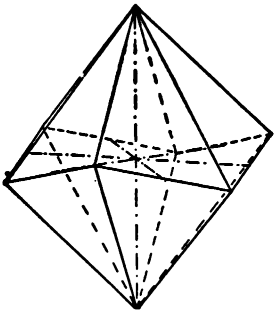
Fig. 184.

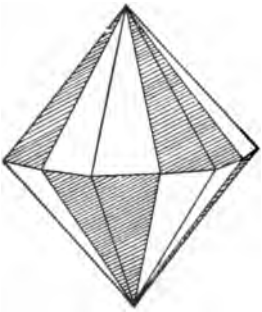
Fig. 185.

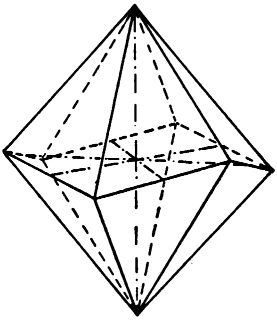
Fig. 186.

The symbols are: $\pm\left[\frac{na : pa : a : mc}{2}\right]$; $+\frac{m\mathrm{P}n}{2}$, $\kappa\{h\,i\,\bar{k}\,l\}$, figure 186; $-\frac{m\mathrm{P}n}{2}$, $\kappa\{i\,h\,\bar{k}\,l\}$, figure 184.

Scalenohedrons with twelve equal polar edges are crystallographically impossible, see page 49.

The axis of trigonal symmetry passes through the two hexahedral angles, while those of binary symmetry bisect the lateral edges.

The other holohedral forms remain unchanged by the application of the rhombohedral hemihedrism, compare figures 178, 182, and 185.

Abbreviated Symbols of Naumann. In order to more easily express some of the interesting relationships existing between forms of this class, which are quite common, Naumann substituted for $\pm\frac{\mathrm{P}}{2}$ and $\pm\frac{m\mathrm{P}}{2}$, the symbols of the rhombohedrons used above, $\pm$R and $\pm$ mR, respectively. For every rhombohedron there exists a series of scalenohedrons, whose lateral edges coincide with those of the rhombohedron, as shown in figure 187. The inscribed rhombohedron is known as "*the rhombohedron of the middle edges.*" The scalenohedrons may, therefore, be indicated in general by mRn, mR representing the "rhombohedron of the middle edges." In mRn, n has reference to the value of the c axis[1] of the scalenohedron in respect to that of the rhombohedron mR. For example, in figure 187, the length of the c axis of the scalenohedron is three times that of the inscribed positive unit rhombohedron. The symbol for this scalenohedron is, hence, according to Naumann +R3.

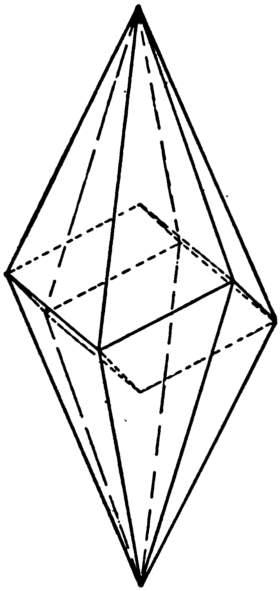

Fig. 187.

The prism of the first order ∞P, also the basal pinacoid OP, are often written ∞R and OR, respectively. The symbols for the other forms mP2, ∞P2, ∞Pn, remain unchanged.

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[1] And not to the secondary axes, as is usually the case in the regular Naumann symbols.

To transform the full into the abbreviated symbols and *vice versa*, the following formulæ are useful:

1. $\frac{mPn}{2} = \frac{m(2-n)}{n} R \frac{n}{2-n}$. Thus, $\frac{4P\frac{4}{3}}{2} = 2R2$.

2. $mRn = \frac{mnP\frac{2n}{n+1}}{2}$. For example, $2R2 = \frac{4P\frac{4}{3}}{2}$.

3. $\kappa\{h\bar{i}\bar{k}l\} = \frac{2k-h}{l} R \frac{h}{2k-h}$. Here, $\kappa\{4\bar{1}\bar{3}1\} = 2R2$.

4. $mRn = \kappa\left\{2n.\ -(n-1).\ -(n+1).\ \frac{2}{m}\right\}$. For instance, $2R2 = \kappa\{4\bar{1}\bar{3}1\}$.

The principal features of the forms of this class are given in the following table:

FORMS	SYMBOLS			Number of Faces	Solid Angles		
	Weiss	Naumann	Miller-Bravais		Trihedral	Tetrahedral	Hexahedral
Rhombohedrons	$\pm\left[\frac{a:\infty a:a:mc}{2}\right]$	$+mR$ $-mR$	$\kappa\{ho\bar{h}l\}$ $\kappa\{oh\bar{h}l\}$	6	2+6	—	—
Hexagonal Bipyramid Second order	$2a:2a:a:mc$	$mP2$	$\kappa\{hh\bar{2}\bar{h}l\}$	Apparently holohedral			
Scalenohedrons	$\pm\left[\frac{na:pa:a:mc}{2}\right]$	$+mRn$ $-mRn$	$\kappa\{hi\bar{k}l\}$ $\kappa\{ih\bar{k}l\}$	12	—	6	2
Hexagonal Prism First order	$a:\infty a:a:\infty c$	∞R	$\kappa\{ho\bar{h}o\}$	Apparently holohedral			
Hexagonal Prism Second order	$2a:2a:a:\infty c$	$\infty P2$	$\kappa\{11\bar{2}0\}$				
Dihexagonal Prism	$na:pa:a:\infty c$	∞Pn	$\kappa\{hi\bar{k}o\}$				
Basal Pinacoid	$\infty a:\infty a:\infty a:c$	OR	$\kappa\{0001\}$				

Combinations. Many of the more common minerals crystallize in this class, for example, Calcite, $CaCO_3$; Hematite, Fe_2O_3; Corundum, Al_2O_3; and Siderite, $FeCO_3$.

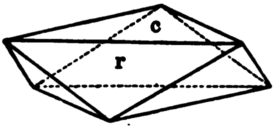

Fig. 188.

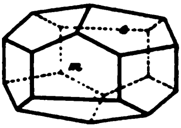

Fig. 189.

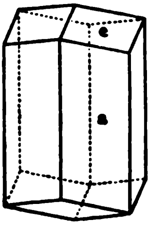

Fig. 190.

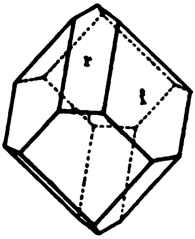

Fig. 191.

Figure 188. $r = R$, $\kappa\{10\bar{1}1\}$; $c = OR$, $\kappa\{0001\}$. Calcite.
Figure 189. $e = -\frac{1}{2}R$, $\kappa\{01\bar{1}2\}$; $m = \infty R$, $\kappa\{10\bar{1}0\}$. Calcite.
Figure 190. $e = -\frac{1}{2}R$, $\kappa\{01\bar{1}2\}$; $a = \infty P2$, $\kappa\{11\bar{2}0\}$. Calcite.
Figure 191. $r = R$, $\kappa\{10\bar{1}1\}$; $f = -2R$, $\kappa\{02\bar{2}1\}$. Calcite.

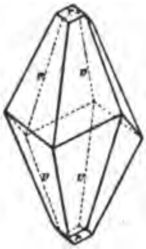

Fig. 192.

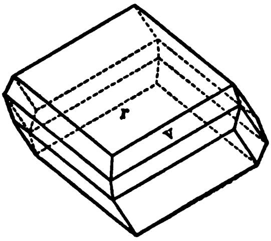

Fig. 193.

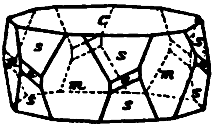

Fig. 194.

Figure 192. $v = R3$, $\kappa\{21\bar{3}1\}$; $r = R$, $\kappa\{10\bar{1}1\}$. Calcite.
Figure 193. $r = R$, $\kappa\{10\bar{1}1\}$; $v = R3$, $\kappa\{21\bar{3}1\}$. Calcite.
Figure 194. $c = OR$, $\kappa\{0001\}$; $m = \infty R$, $\kappa\{10\bar{1}0\}$; $s = R5$, $\kappa\{32\bar{5}1\}$; $a = \infty P2$, $\kappa\{11\bar{2}0\}$. Calcite.

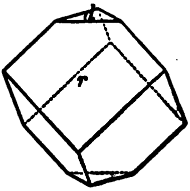

Fig. 195.

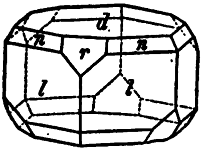

Fig. 196.

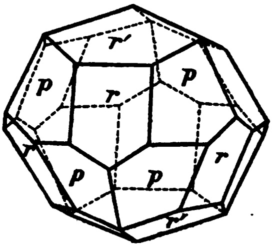

Fig. 197.

Figure 195. d = OR, $\kappa\{0001\}$; r = R, $\kappa\{10\bar{1}1\}$. Corundum.

Figure 196. d = OR, $\kappa\{0001\}$; r = R, $\kappa\{10\bar{1}1\}$; $n = \frac{4}{3}$P2, $\kappa\{22\bar{4}3\}$; $l = \infty$P2, $\kappa\{11\bar{2}0\}$. Corundum.

Figure 197. r = R, $\kappa\{10\bar{1}1\}$; $r' = \frac{1}{4}$R, $\kappa\{10\bar{1}4\}$; $p = \frac{4}{3}$P2, $\kappa\{22\bar{4}3\}$. Hematite.

Figure 198. r = R, $\kappa\{10\bar{1}1\}$; $4r$ = 4R, $\kappa\{40\bar{4}1\}$; w = R5, $\kappa\{32\bar{5}1\}$; z = R3, $\kappa\{21\bar{3}1\}$; $g = \infty$R, $\kappa\{10\bar{1}0\}$. Calcite.

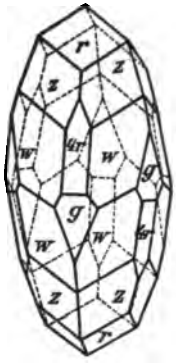

Fig. 198.

6. *HEXAGONAL BIPYRAMIDAL CLASS.*[1]

(Pyramidal Hemihedrism.)

Symmetry. The secondary and intermediate planes and the six axes of binary symmetry are lost. The remaining elements are the principal plane, the hexagonal axis and the center of symmetry, figure 203, page 66.

Hexagonal bipyramids of the third order. Figure 200 illustrates the pyramidal method of extension and suppression of faces on the dihexagonal bipyramid. It is obvious that this bipyramid yields two new forms, each bounded by twelve equal isosceles triangles. They are termed *positive* (figure 201) and *negative*[2] (figure 199) *hexagonal bipyramids of the third order*. In form these bipyramids do not differ from those of the first and second orders. Figures 202 and 204 show the positions of these bipyramids in respect to the secondary axes. The inner and outer light outlines represent the horizontal cross-sections of the bipyramids of the *first* and *second* orders, respectively, while the heavy outlines show the cross-sections of those of the third order. These occupy an intermediate position.

The symbols are: $\pm\left[\frac{na:pa:a:mc}{2}\right]$; $+\frac{mPn}{2}$, $\pi\{hi\bar{k}l\}$ figure 201; $-\frac{mPn}{2}$, $\pi\{ih\bar{k}l\}$ figure 199.

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[1] Called by Dana the *pyramidal group*.

[2] The terms *right* and *left* are sometimes used.

The axis of hexagonal symmetry passes through the hexahedral angles. The position of the secondary crystallographic axes is shown by figures 202 and 204. These do not join the tetrahedral angles or the centers of the basal edges, but some point between them, which is dependent upon the value of *n*. Compare figures 135 to 136.

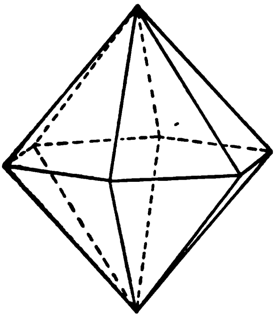
Fig. 199.

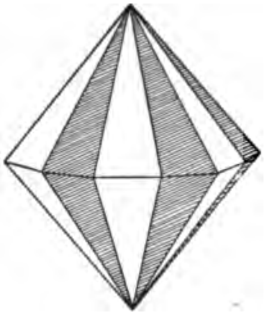
Fig. 200.

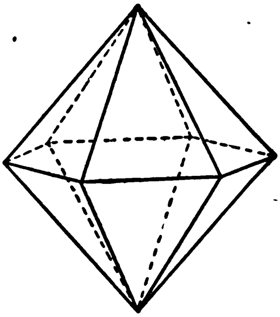
Fig. 201.

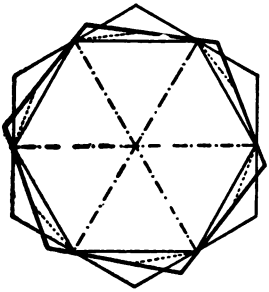
Fig. 202.

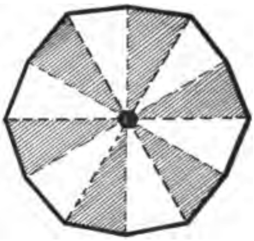
Fig. 203.

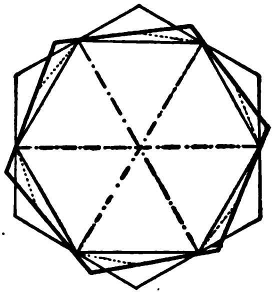
Fig. 204.

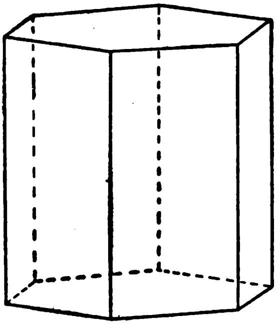
Fig. 205.

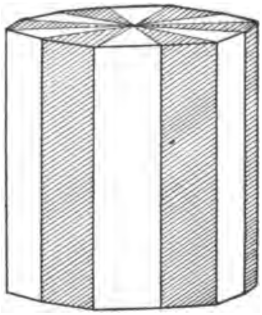
Fig. 206.

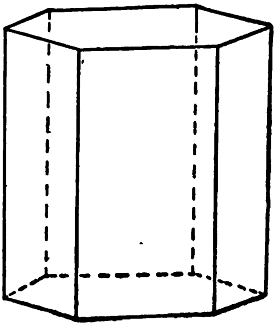
Fig. 207.

Hexagonal prisms of the third order. From the dihexagonal prism two congruent forms consisting of six planes, the *positive* (figure 207) and *negative* (figure 205) *hexagonal prisms of the third order* are derived. Figures 202 and 204 show these forms and their relation to the other hexagonal prisms. Their position in regard to the secondary axes is the same as for the bipyramids of this order.

The symbols are: $\pm\left[\frac{na : pa : a : \infty c}{2}\right]$; $+\frac{\infty P n}{2}$, $\pi\{hi\bar{k}o\}$ figure 207; $-\frac{\infty P n}{2}$, $\pi\{ih\bar{k}o\}$ figure 205.

The axis of hexagonal symmetry extends parallel to the edges.

The other holohedral forms are unaltered by the application of the pyramidal hemihedrism, since the suppression effects only one half of each face. Compare figures 200, 203 and 206. They are, hence, apparently holohedral.

The principal features of this class have been summarized in the following table:

FORMS	SYMBOLS			Number of Faces	Solid Angles	
	Weiss	Naumann	Miller-Bravais		Tetrahedral	Hexahedral
Hexagonal Bipyramids First order	$a : \infty a : a : mc$	mP	$\pi\{h\,o\,\bar{h}\,l\}$	Apparently holohedral		
Hexagonal Bipyramids Second order	$2a : 2a : a : mc$	$mP2$	$\pi\{h\,h\,\overline{2h}\,l\}$			
Hexagonal Bipyramids Third order	$\pm\left[\frac{na : pa : a : mc}{2}\right]$	$+\frac{mPn}{2}$ $-\frac{mPn}{2}$	$\pi\{h\,i\,\bar{k}\,l\}$ $\pi\{i\,h\,\bar{k}\,l\}$	12	6	2
Hexagonal Prism First order	$a : \infty a : a : \infty c$	∞P	$\pi\{h\,o\,\bar{h}\,o\}$	Apparently holohedral		
Hexagonal Prism Second order	$2a : 2a : a : \infty c$	$\infty P2$	$\pi\{1\,1\,\bar{2}\,0\}$			
Hexagonal Prisms Third order	$\pm\left[\frac{na : pa : a : \infty c}{2}\right]$	$+\frac{\infty Pn}{2}$ $-\frac{\infty Pn}{2}$	$\pi\{h\,i\,\bar{k}\,o\}$ $\pi\{i\,h\,\bar{k}\,o\}$	6	—	—
Basal Pinacoid	$\infty a : \infty a : \infty a : c$	OP	$\pi\{0\,0\,0\,1\}$	Apparently holohedral		

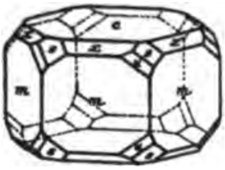

Fig. 208.

Combinations. Figure 208. $m = \infty P$, $\pi\{10\bar{1}0\}$; $c = OP$, $\pi\{0001\}$; $x = P$, $\pi\{10\bar{1}1\}$; $s = 2P2$, $\pi\{11\bar{2}1\}$; $u = +\frac{3P\frac{3}{2}}{2}$, $\pi\{12\bar{3}1\}$. This combination has been observed on apatite, $Ca_5Cl(PO_4)_3$.

6. HEXAGONAL TRAPEZOHEDRAL CLASS.[1]

(*Trapezohedral Hemihedrism.*)

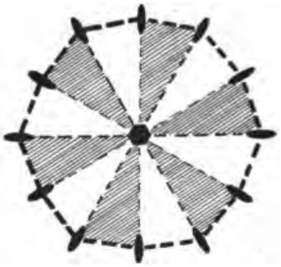

Fig. 209.

Symmetry. The center and all planes of symmetry disappear. Hence, the elements of this class are the hexagonal axis and six binary axes of symmetry, figure 209.

Hexagonal Trapezohedrons. From figure 211 it is evident that by means of this hemihedrism, the dihexagonal bipyramid yields two correlated forms bounded by similar trapeziums. The forms are enantiomorphous and designated as the *right* and *left hexagonal trapezohedrons.*

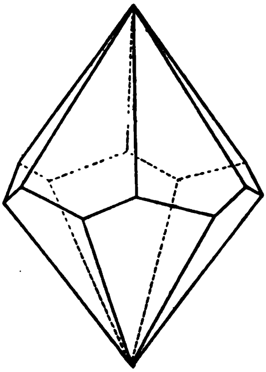

Fig. 210.

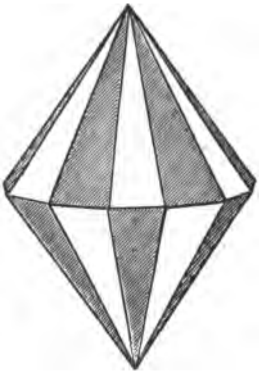

Fig. 211.

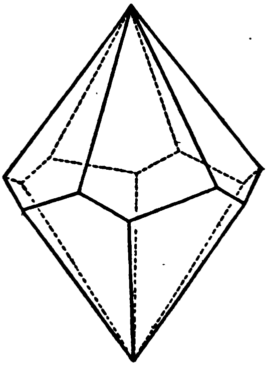

Fig. 212.

Their symbols are: $r, l \left[\frac{na : pa : a : mc}{2}\right]$; $r\frac{mPn}{2}$, $\tau\{hi\bar{k}l\}$ figure 212; $l\frac{mPn}{2}$, $\tau\{k\bar{i}\bar{h}l\}$ figure 210.

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[1] *Trapezohedral group* of Dana.

On the *right* trapezohedrons the six longer basal edges are inclined to the left of the observer and, vice versa, to the right in the case of the *left* form, compare figures 212 and 210.

The axis of hexagonal symmetry joins the hexahedral angles. The secondary crystallographic axes connect the centers of the longer basal edges, the intermediate those of the shorter. These axes possess binary symmetry.

Since this hemihedrism calls for the suppression of only portions of each of the faces of the other holohedrons, no new forms can be derived from them. They are apparently holohedral.

The chief features of the forms of this class are given in the following table:

FORMS	SYMBOLS			Number of Faces	Solid Angles	
	Weiss	Naumann	Miller-Bravais		Trihedral	Hexahedral
Hexagonal Bipyramids First order	$a : \infty a : a : mc$	mP	$\tau\{h o \bar{h} l\}$	Apparently holohedral		
Hexagonal Bipyramids Second order	$2a : 2a : a : mc$	$mP2$	$\tau\{h h \overline{2h} l\}$			
Hexagonal Trapezohedrons	$r, l \left[\frac{na : pa : a : mc}{2}\right]$	$r\,\frac{mPn}{2}$ $l\,\frac{mPn}{2}$	$\tau\{h i \bar{k} l\}$ $\tau\{k \bar{i} \bar{h} l\}$	12	12	2
Hexagonal Prism First order	$a : \infty a : a : \infty c$	∞P	$\tau\{h o \bar{h} o\}$	Apparently holohedral		
Hexagonal Prism Second order	$2a : 2a : a : \infty c$	$\infty P2$	$\tau\{1 1 \bar{2} 0\}$			
Dihexagonal Prism	$na : pa : a : \infty c$	∞Pn	$\tau\{h i \bar{k} o\}$			
Basal Pinacoid	$\infty a : \infty a : \infty a : c$	OP	$\tau\{0001\}$			

No crystals showing the occurrence of the hexagonal trapezohedrons have as yet been discovered. The double salt, barium stibiotartrate and potassium nitrate, $Ba(SbO)_2 (C_4H_4O_6)_2 + KNO_3$, and the corresponding lead salt are assigned to this class. The crystals are apparently holohedral, the etch figures, however, reveal the lower grade of symmetry.

7. DITRIGONAL PYRAMIDAL CLASS.[1]

(*Trigonal Hemihedrism with Hemimorphism.*)

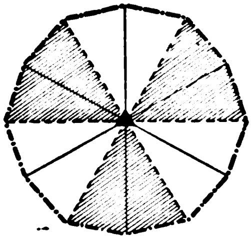

Fig. 213.

Symmetry. If the forms of the ditrigonal bipyramidal class, page 56, become hemimorphic, the principal plane of symmetry and, of course, the axes of binary symmetry also are lost. Three intermediate planes and a polar axis of trigonal symmetry parallel to the *c* axis are the only elements left. These symmetry relations are shown in figure 213.

The forms of this class may, moreover, be considered also as derived from those of the ditrigonal scalenohedral class, that is, from those showing rhombohedral hemihedrism, page 60. A comparison of figures 159 and 178 reveals the fact that when hemimorphism becomes effective the resulting forms must in both cases be the same. Therefore, this class is often known as the *rhombohedral hemimorphic* class.

Trigonal pyramid of the first order. The trigonal bipyramid, page 56, the result of the application of the trigonal hemihedrism to the hexagonal bipyramid of the first order, yields upon the addition of hemimorphism *upper* and *lower* forms, consisting of but three faces each. Since there are two trigonal bipyramids, one positive and the other negative, it follows that each hexagonal bipyramid of the first order now yields four forms.[2] Being singly terminated, they are termed *trigonal* pyramids.[3]

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[1] Called by Dana the *rhombohedral hemimorphic* class.

[2] The forms are, hence, sometimes said to be the result of tetartohedrism, namely the ***ditrigonal pyramidal tetartohedrism***.

[3] Compare footnote on page 54. The letters *u* and *l* designate the upper and lower forms, respectively.

Their symbols are:

Positive upper, $+\left[\frac{a : \infty a : a : mc}{4}\right] u, \quad \frac{m\mathrm{P}}{4} u, \ \{h\,\mathrm{o}\,\bar{h}\,l\}.$

Positive lower, $+\left[\frac{a : \infty a : a : mc}{4}\right] l, + \frac{m\mathrm{P}}{4} l, \ \{h\,\mathrm{o}\,\bar{h}\,\bar{l}\}.$

Negative upper, $-\left[\frac{a : \infty a : a : mc}{4}\right] u, - \frac{m\mathrm{P}}{4} u, \ \{\mathrm{o}\,h\,\bar{h}\,l\}.$

Negative lower, $-\left[\frac{a : \infty a : a : mc}{4}\right] l, - \frac{m\mathrm{P}}{4} l, \ \{\mathrm{o}\,h\,\bar{h}\,\bar{l}\}.$

Hexagonal pyramids of the second order. The hexagonal bipyramid of this order remains unaffected by the trigonal hemihedrism, page 59, but now yields an *upper* and *lower* form, each consisting of six faces, termed *hexagonal pyramids of the second order.*

The symbols are: $\left[\frac{2a : 2a : a : mc}{2}\right] u, \ l; \ \frac{m\mathrm{P}2}{2} u, \ \{h\,h\,\bar{2}\bar{h}\,l\},$ $\frac{m\mathrm{P}2}{2} l, \ \{h\,h\,\bar{2}\bar{h}\,\bar{l}\}.$

Ditrigonal pyramids. The dihexagonal bipyramid by means of the trigonal hemihedrism yields the positive and negative ditrigonal bipyramids, page 57. Each of these bipyramids now gives rise to an upper and lower form, termed *ditrigonal* pyramids.[1] These, like the preceding pyramids of this class, are *open* forms. They consist of six faces.

The symbols are:

Positive upper, $+\left[\frac{na : pa : a : mc}{4}\right] u, \ + \frac{m\mathrm{P}n}{4} u, \ \{h\,i\,\bar{k}\,l\}.$

Positive lower, $+\left[\frac{na : pa : a : mc}{4}\right] l, \ + \frac{m\mathrm{P}n}{4} l, \ \{h\,i\,\bar{k}\,\bar{l}\}.$

---

[1] Since, as said on page 70, these forms may be considered as derived from those showing rhombohedral hemihedrism, the terms *rhombohedron-* and *scalenohedron-like* faces are sometimes used for the trigonal and ditrigonal pyramids, respectively, the forms being assumed to be half of the rhombohedron and scalenohedron.

Negative upper, $-\left[\frac{na : pa : a : mc}{4}\right] u, \quad -\frac{mPn}{4} u, \quad \{i h \bar{k} l\}$.

Negative lower, $-\left[\frac{na : pa : a : mc}{4}\right] l, \quad -\frac{mPn}{4} l, \quad \{i h \bar{k} \bar{l}\}$.

Trigonal prisms. Hemimorphism does not affect, morphologically, the trigonal prisms, the hemihedrons of the hexagonal prism of the first order, page 56, for each face belongs alike to the upper and lower poles.

The symbols are: $\pm\left[\frac{a : \infty a : a : \infty c}{2}\right]; \ +\frac{\infty P}{2}, \ \{h 0 \bar{h} 0\}$; $-\frac{\infty P}{2}, \ \{0 h \bar{h} 0\}$.

Hexagonal prism of the second order. It will be recalled that the hexagonal prism of this order remains unchanged, geometrically, by the trigonal hemihedrism, page 59. Hence, as in the preceding case, each face belongs alike to the upper and lower poles, hemimorphism also can not be effective in the production of new forms. This prism is, therefore, still apparently holohedral.

Its symbols are: $2a : 2a : a : \infty c, \ \infty P2, \ \{h h 2\bar{h} 0\}$.

Ditrigonal prisms. As in the case of the trigonal prisms, page 59, these forms remain unchanged.

The symbols are: $\pm\left[\frac{na : pa : a : \infty c}{2}\right]; \ +\frac{\infty Pn}{2}, \ \{h i \bar{k} 0\}$; $-\frac{\infty Pn}{2}, \ \{i h \bar{k} 0\}$.

These, together with the trigonal prisms, are not to be distinguished morphologically from those of the ditrigonal bipyramidal class. They are apparently hemihedral.

Basal pinacoids. This form now evidently consists of two types, *upper* and *lower*, of one face each.

The symbols are: $\left[\frac{\infty a : \infty a : \infty a : c}{2}\right] u, l; \ \frac{0P}{2} u, \ \{0001\}$; $\frac{0P}{2} l, \ \{000\bar{1}\}$.

In all the above forms the crystallographic axes are located as in the ditrigonal bipyramidal class.

The following table shows the principal features of the forms of this class:

FORMS	SYMBOLS			Number of Faces	Solid Angles	
	Weiss	Naumann	Miller-Bravais		Trihedral	Hexahedral
Trigonal Pyramids First order	$\pm u, \pm l \left\{\frac{a:\infty a:a:mc}{4}\right\}$	$+\frac{mP}{4}u$ $+\frac{mP}{4}l$ $-\frac{mP}{4}u$ $-\frac{mP}{4}l$	$\{h0\bar{h}l\}$ $\{h0\bar{h}\bar{l}\}$ $\{0h\bar{h}l\}$ $\{0h\bar{h}\bar{l}\}$	3	1	–
Hexagonal Pyramids Second order	$u, l \left\{\frac{2a:2a:a:mc}{2}\right\}$	$\frac{mP2}{2}u$ $\frac{mP2}{2}l$	$\{hh\bar{2h}l\}$ $\{hh\bar{2h}\bar{l}\}$	6	–	1
Ditrigonal Pyramids	$\pm u, \pm l \left\{\frac{na:pa:a:mc}{4}\right\}$	$+\frac{mPn}{4}u$ $+\frac{mPn}{4}l$ $-\frac{mPn}{4}u$ $-\frac{mPn}{4}l$	$\{hi\bar{k}l\}$ $\{hi\bar{k}\bar{l}\}$ $\{ih\bar{k}l\}$ $\{ih\bar{k}\bar{l}\}$	6	–	1
Trigonal Prisms	$\pm \left\{\frac{a:\infty a:a:\infty c}{2}\right\}$	$+\frac{\infty P}{2}$ $-\frac{\infty P}{2}$	$\{h0\bar{h}0\}$ $\{0h\bar{h}0\}$	Apparently of ditrigonal bipyramidal class.		
Ditrigonal Prisms	$\pm \left\{\frac{na:pa:a:mc}{2}\right\}$	$+\frac{\infty Pn}{2}$ $-\frac{\infty Pn}{2}$	$\{hi\bar{k}0\}$ $\{ih\bar{k}0\}$			
Hexagonal Prism Second order	$2a:2a:a:\infty c$	$\infty P2$	$\{hh\bar{2h}0\}$	Apparently holohedral		
Basal Pinacoids	$u, l \left\{\frac{\infty a:\infty a:\infty a:c}{2}\right\}$	$\frac{OP}{2}u$ $\frac{OP}{2}l$	$\{0001\}$ $\{000\bar{1}\}$	1	–	–

Combinations. The mineral tourmaline furnishes excellent combinations of the above forms.

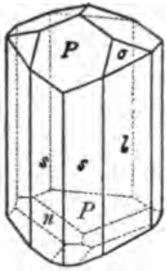

Fig. 214.

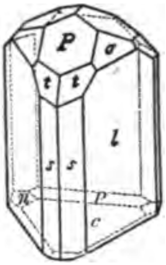

Fig. 215.

In the accompanying figures 214 and 215, $\mathrm{P} = +\frac{\mathrm{P}}{4}u$, $\{10\bar{1}1\}$; $P = +\frac{\mathrm{P}}{4}l$, $\{01\bar{1}\bar{1}\}$; $o = \frac{2\mathrm{P}}{4}u$, $\{02\bar{2}1\}$; $c = \frac{\mathrm{OP}}{2}l$, $\{000\bar{1}\}$; $n = +\frac{\frac{1}{2}\mathrm{P}}{4}l$, $\{01\bar{1}\bar{2}\}$; $s = \infty\mathrm{P}2$, $\{11\bar{2}0\}$; $t = +\frac{3\mathrm{P}\frac{3}{2}}{4}u$, $21\bar{3}1$; $l = -\frac{\infty\mathrm{P}}{2}$, $\{01\bar{1}0\}$.

As indicated on page 70, these forms may be assumed as derived from those of the ditrigonal scalenohedral class, *i. e.*, showing rhombohedral hemihedrism, and, hence, are often designated as follows:

$$\mathrm{P} = +\frac{\mathrm{R}}{2}u;\ P = +\frac{\mathrm{R}}{2}l;\ n = -\tfrac{1}{2}\mathrm{R}l;\ t = \frac{\mathrm{R}3}{2}u;\ c = \frac{\mathrm{OR}}{2}l;$$

$$s = \infty\mathrm{P}2;\ l = -\frac{\infty\mathrm{R}}{2};\ o = -2\mathrm{R}u.$$

8. HEXAGONAL PYRAMIDAL CLASS.

(*Pyramidal Hemihedrism with Hemimorphism.*)

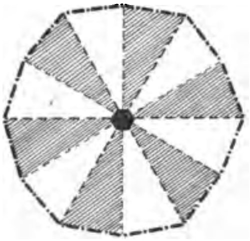

Fig. 216.

Symmetry. This class possesses but one element of symmetry, namely, a polar axis of hexagonal symmetry, as shown in figure 216. This is obvious, for when the forms of the hexagonal bipyramidal class, page 65, become hemimorphic, the principal plane and center of symmetry are of necessity lost. Forms of exactly the same character are, moreover, to be derived by applying hemimorphism to those of the hexagonal trapezohedral class. Compare figures 203 and 209.

Forms. The bipyramids of the first, second and third orders of the hexagonal bipyramidal class[1] now consist of *upper* and *lower* pyramids. The prisms, on the other hand, remain unaffected morphologically. The basal pinacoid, of course, is now composed of an upper and lower form.

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[1] If, however, hemimorphism be applied to the hexagonal trapezohedral forms, the hexagonal trapezohedrons and dihexagonal prism would yield forms corresponding precisely to the pyramids and prisms of the third order. The other forms being apparently holohedral in both classes would, of course, be affected alike by the introduction of hemimorphism.

As in the preceding class, these are now *open* forms. Their position in respect to the crystallographic axes is the same as in the hexagonal bipyramidal class, page 65.

The principal features are given in the following table:

FORMS	SYMBOLS			FACES
	Weiss	Naumann	Miller-Bravais	
Hexagonal Pyramids First order	$u, l\left[\frac{a : \infty a : a : mc}{2}\right]$	$\frac{mP}{2}u$ $\frac{mP}{2}l$	$\{h o \bar{h} l\}$ $\{h o \bar{h} \bar{l}\}$	6
Hexagonal Pyramids Second order	$u, l\left[\frac{2a : 2a : a : mc}{2}\right]$	$\frac{mP2}{2}u$ $\frac{mP2}{2}l$	$\{h h \bar{2h} l\}$ $\{h h \bar{2} \bar{h} \bar{l}\}$	6
Hexagonal Pyramids Third order	$\pm u, \pm l\left[\frac{na : pa : a : mc}{4}\right]$	$+\frac{mPn}{4}u$ $+\frac{mPn}{4}l$ $-\frac{mPn}{4}u$ $-\frac{mPn}{4}l$	$\{h i \bar{k} l\}$ $\{h i \bar{k} \bar{l}\}$ $\{k \bar{i} \bar{h} l\}$ $\{k \bar{i} \bar{h} \bar{l}\}$	6
Hexagonal Prism First order	$a : \infty a : a : \infty c$	∞P	$\{h o \bar{h} o\}$	Apparently holohedral
Hexagonal Prisms Second order	$2a : 2a : a : \infty c$	$\infty P2$	$\{h h \bar{2} \bar{h} l\}$	
Hexagonal Prisms Third order	$\pm\left[\frac{na : pa : a : \infty c}{2}\right]$	$+\frac{\infty Pn}{2}$ $-\frac{\infty Pn}{2}$	$\{h i \bar{k} o\}$ $\{k \bar{i} \bar{h} o\}$	6
Basal Pinacoids	$u, l\left[\frac{\infty a : \infty a : \infty a : c}{2}\right]$	$\frac{OP}{2}u$ $\frac{OP}{2}l$	$\{0001\}$ $\{000\bar{1}\}$	1

Combinations. By means of etch figures crystals of a number of compounds have been referred to this class. Among the minerals,

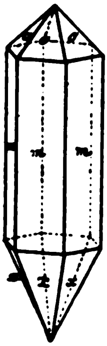

Fig. 217.

nepheline, $Na_8Al_8Si_9O_{34}$, is to be mentioned. Figure 217 shows a crystal of strontium antimonyltartrate, $Sr(SbO)_2(C_4H_4O_6)_2$, $m = \infty P$, $\{1010\}$; $o = \frac{P}{2} u$, $\{10\bar{1}1\}$; $x = \frac{2P}{2} l$, $\{20\bar{2}\bar{1}\}$.

HEXAGONAL TETARTOHEDRISMS.

The tetartohedral forms may be assumed as derived from the holohedral by applying simultaneously two types of hemihedrism, page 55. Thus, by combining the trigonal and pyramidal hemihedrisms, the *trigonal tetartohedrism* results, figure 218. A second type, known as the *trapezohedral tetartohedrism*, is the result of the combination of the rhombohedral and trapezohedral hemihedrism, figure 219. The simultaneous application of the rhombohedral and pyramidal hemihedrisms gives rise to the *rhombohedral tetartohedrism*, figure 220.

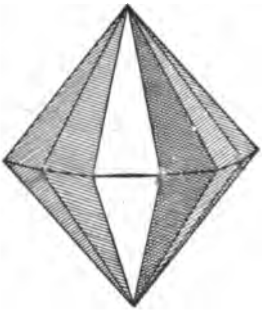

Fig. 218.

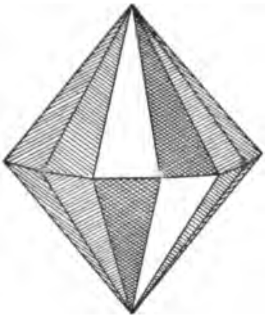

Fig. 219.

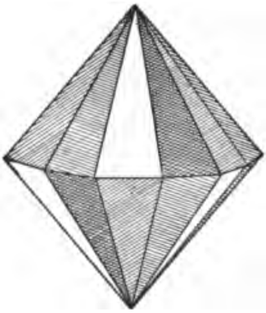

Fig. 220.

These three are the only types possible, for other combinations of the four methods of hemihedrisms do not satisfy the conditions of tetartohedrism.

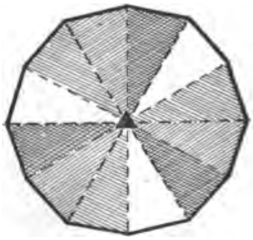

Fig. 221

9. TRIGONAL BIPYRAMIDAL CLASS.

(*Trigonal Tetartohedrism.*)

Symmetry. This class possesses two elements of symmetry, namely, a principal plane and an axis of trigonal symmetry, figure 221.

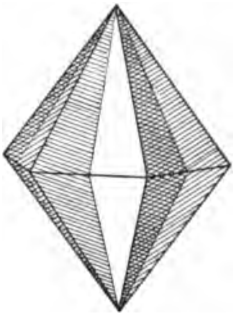
Fig. 222.

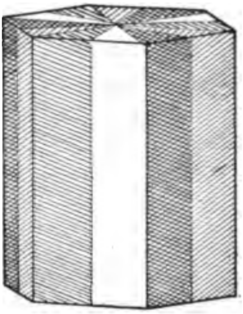
Fig. 223.

Trigonal bipyramids and prisms of the first order. As can be readily seen from figures 222 and 223, the hexagonal bipyramids and prism of the first order now yield trigonal bipyramids and prisms, respectively. These forms do not differ morphologically from those of the ditrigonal bipyramidal class, compare figures 160 to 168.

The symbols for the trigonal bipyramids of the first order are:

$$+\left[\frac{a:\infty a:a:mc}{2}\right],\ +\frac{mP}{2},\ \{h\,o\,\bar{h}\,l\};\ \text{and}$$

$$-\left[\frac{a:\infty a:a:mc}{2}\right],\ -\frac{mP}{2},\ \{o\,h\,\bar{h}\,l\}.$$

Those for the corresponding trigonal prisms being:

$$+\left[\frac{a:\infty a:a:\infty c}{2}\right],\ +\frac{\infty P}{2},\ \{h\,o\,\bar{h}\,o\};\ \text{and}$$

$$-\left[\frac{a:\infty a:a:\infty c}{2}\right],\ -\frac{\infty P}{2},\ \{o\,h\,\bar{h}\,o\}.$$

Trigonal bipyramids and prisms of the second order. These forms are obtained from the hexagonal bipyramid and prism of the

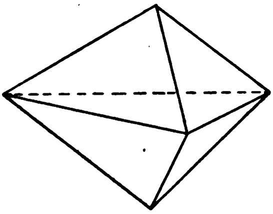
Fig. 224.

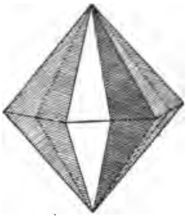
Fig. 225.

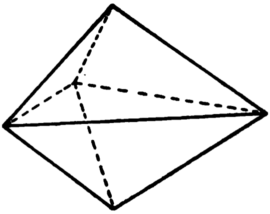
Fig. 226.

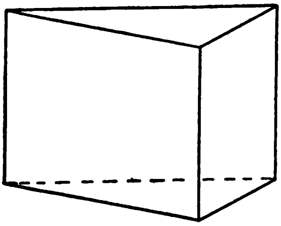
Fig. 227.

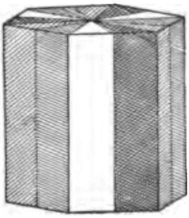
Fig. 228.

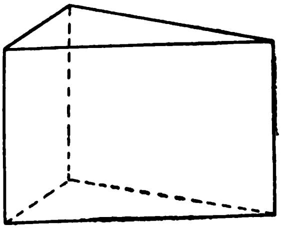
Fig. 229.

second order as can be seen from figures 224 to 229. The position of these forms as well as those of the other two orders in respect to the crystallographic axes is shown in figure 237.

The symbols for the trigonal bipyramids of the second order:

$+\left[\frac{2a : 2a : a : mc}{2}\right]$, $+\frac{mP2}{2}$, $\{h\,h\,\bar{2}\,\bar{h}\,l\}$, figure 226; and

$-\left[\frac{2a : 2a : a : mc}{2}\right]$, $-\frac{mP2}{2}$, $\{2\,h\,\bar{h}\,\bar{h}\,l\}$, figure 224.

Those of the corresponding trigonal prisms:

$+\left[\frac{2a : 2a : a : \infty c}{2}\right]$, $+\frac{\infty P2}{2}$, $\{h\,h\,\bar{2}\,\bar{h}\,0\}$, figure 229, and

$-\left[\frac{2a : 2a : a : \infty c}{2}\right]$, $-\frac{\infty P2}{2}$, $\{2\,h\,\bar{h}\,\bar{h}\,0\}$, figure 227.

Trigonal bipyramids and prisms of the third order. The dihexagonal bipyramid now furnishes four new forms, termed *trigonal bipyramids of the third order*, while the dihexagonal prism yields the corresponding prisms. Figures 230 to 235 show the derivation of two forms of the bipyramid and prism. Figures 236 and 237, however, show the position of these forms in respect to the crystallographic axes as also to the forms of the other orders.

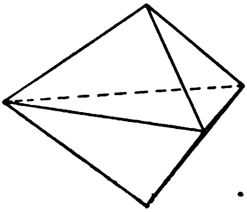
Fig. 230.

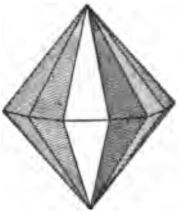
Fig. 231.

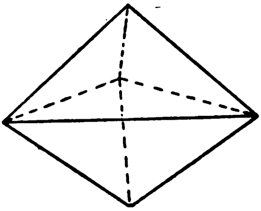
Fig. 232.

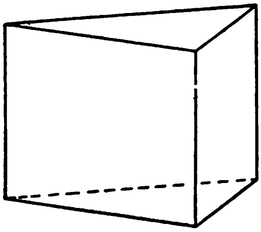
Fig. 233.

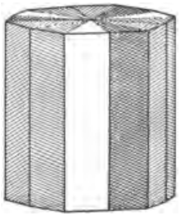
Fig. 234.

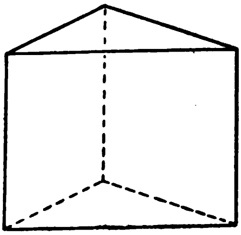
Fig. 235.

The symbols of the bipyramids are:

Positive right,

$+r\left[\frac{na : pa : a : mc}{2}\right]$; $+r\frac{mPn}{4}$; $\{h\,i\,\bar{k}\,l\}$, figure 232.

Positive left,

$+l\left[\frac{na : pa : a : mc}{2}\right]$; $+l\frac{mPn}{4}$; $\{k\,\bar{i}\,\bar{h}\,l\}$, figure 230.

Negative right,

$-r\left[\frac{na : pa : a : mc}{2}\right]$; $-r\frac{mPn}{4}$; $\{i\,h\,\bar{k}\,l\}$.

Negative left,

$-l\left[\frac{na : pa : a : mc}{2}\right]$; $-l\frac{mPn}{4}$; $\{k\,\bar{h}\,\bar{i}\,l\}$.

Those of the prisms are:

$+r\left[\frac{na : pa : a : \infty c}{2}\right]$; $+r\frac{\infty Pn}{4}$; $\{h\,i\,\bar{k}\,o\}$, figure 235.

$+l\left[\frac{na : pa : a : \infty c}{2}\right]$; $+l\frac{\infty Pn}{4}$; $\{k\,\bar{i}\,\bar{h}\,o\}$, figure 233.

$-r\left[\frac{na : pa : a : \infty c}{2}\right]$; $-r\frac{\infty Pn}{4}$; $\{i\,h\,\bar{k}\,o\}$.

$-l\left[\frac{na : pa : a : \infty c}{2}\right]$; $-l\frac{\infty Pn}{4}$; $\{k\,\bar{h}\,\bar{i}\,o\}$.

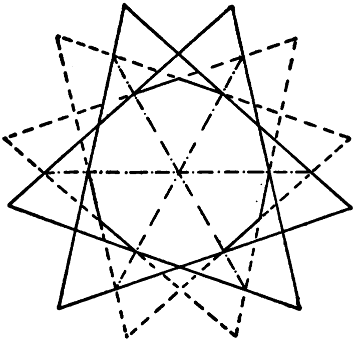

Fig. 236.

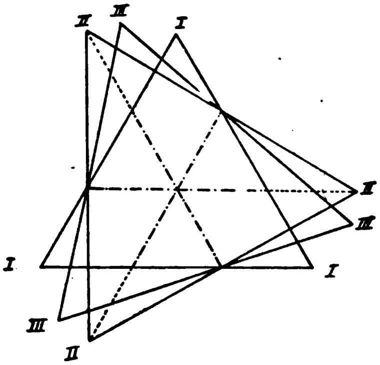

Fig. 237.

The basal pinacoid, as is obvious from figures 223, 228 and 234, remains unaltered.

The following table summarizes the principal features of each form.

No representative of this class has yet been discovered.

FORMS	SYMBOLS			Number of Faces	Solid Angles	
	Weiss	Naumann	Miller-Bravais		Trihedral	Tetrahedral
Trigonal Bipyramids First order	$\pm\left[\frac{a : \infty a : a : mc}{2}\right]$	$+\frac{mP}{2}$ $-\frac{mP}{2}$	$\{h\,0\,\bar{h}\,l\}$ $\{0\,h\,\bar{h}\,l\}$	6	2	3
Trigonal Bipyramids Second order	$\pm\left[\frac{2a : 2a : a : mc}{2}\right]$	$+\frac{mP2}{2}$ $-\frac{mP2}{2}$	$\{h\,h\,\overline{2h}\,l\}$ $\{2h\,\bar{h}\,\bar{h}\,l\}$	6	2	3
Trigonal Bipyramids Third order	$\pm r, \pm l\left[\frac{na : pa : a : mc}{4}\right]$	$+r\frac{mPn}{4}$ $+l\frac{mPn}{4}$ $-r\frac{mPn}{4}$ $-l\frac{mPn}{4}$	$\{h\,i\,\bar{k}\,l\}$ $\{k\,\bar{i}\,\bar{h}\,l\}$ $\{i\,h\,\bar{k}\,l\}$ $\{k\,\bar{h}\,\bar{i}\,l\}$	6	2	3
Trigonal Prisms First order	$\pm\left[\frac{a : \infty a : a : \infty c}{2}\right]$	$+\frac{\infty P}{2}$ $-\frac{\infty P}{2}$	$\{h\,0\,\bar{h}\,0\}$ $\{0\,h\,\bar{h}\,0\}$	3	-	-
Trigonal Prisms Second order	$\pm\left[\frac{2a : 2a : a : \infty c}{2}\right]$	$+\frac{\infty P2}{2}$ $-\frac{\infty P2}{2}$	$\{1\,1\,\bar{2}\,0\}$ $\{2\,\bar{1}\,\bar{1}\,0\}$	3		
Trigonal Prisms Third order	$\pm r, \pm l\left[\frac{na : pa : a : \infty c}{4}\right]$	$+r\frac{\infty Pn}{4}$ $+l\frac{\infty Pn}{4}$ $-r\frac{\infty Pn}{4}$ $-l\frac{\infty Pn}{4}$	$\{h\,i\,\bar{k}\,0\}$ $\{k\,\bar{i}\,\bar{h}\,0\}$ $\{i\,h\,\bar{k}\,0\}$ $\{k\,\bar{h}\,\bar{i}\,0\}$	3	-	-
Basal Pinacoid	$\infty a : \infty a : \infty a : c$	$0P$	$\{0\,0\,0\,1\}$	2	-	-

10. TRIGONAL TRAPEZOHEDRAL CLASS.[1]

(*Trapezohedral Tetartohedrism.*)

Symmetry. A principal axis of trigonal symmetry and three polar axes of binary symmetry are the only elements in this class, figure 238.

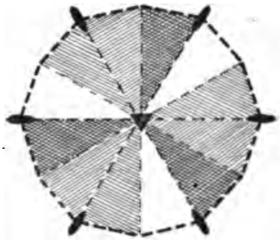

Fig. 238.

Rhombohedrons of the first order. From the hexagonal bipyramid of the first order, two rhombohedrons of the same order result as can be seen from figure 239. These forms are identical, morphologically, with those of the ditrigonal scalenohedral class, page 60.

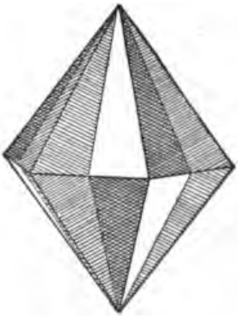

Fig. 239.

Their symbols are: $\pm\left[\frac{a:\infty a:a:mc}{2}\right]$;

$+mR$, $\{h0\bar{h}l\}$; $-mR$, $\{0h\bar{h}l\}$.[2]

Trigonal bipyramids of the second order. The hexagonal bipyramids of the second order, figure 240, yield two trigonal bipyramids of this order. They are identical, geometrically, with those of the trigonal bipyramidal class, page 77.[3]

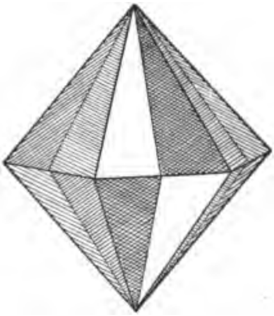

Fig. 240.

The symbols are: $\pm\left[\frac{2a:2a:a:mc}{2}\right]$;

$+\frac{mP2}{2}$, $\{hh\bar{2}\bar{h}l\}$; $-\frac{mP2}{2}$, $\{2h\bar{h}\bar{h}l\}$.

Trigonal Trapezohedrons. The dihexagonal bipyramids yield four new forms, each bounded by six faces, which in the ideal development are equal trapeziums. Figure 242 shows the application of the trapezohedral tetartohedrism so as to give rise to the *positive right trigonal trapezohedron*, figure 243. Figure 241 shows the positive left form. These are *enantiomorphous*.

---

[1] The *trapezohedral group* of Dana.

[2] Sometimes the two Greek letters κ τ are placed before the Miller indices. See footnote on page 40.

[3] Sometimes designated as *right* and *left*.

The trigonal trapezohedrons may also be conceived as derived from the scalenohedron by the subsequent application of hemihedrism. This is shown by figure 245, a negative scalenohedron, which now yields the *negative right* (figure 246) and *negative left* (figure 244) *trigonal trapezohedrons*, respectively.

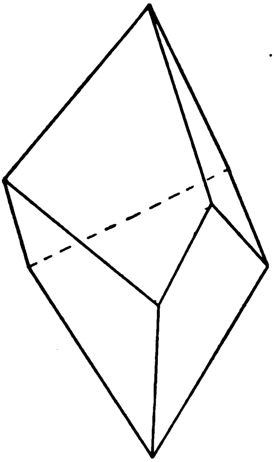
Fig. 241.

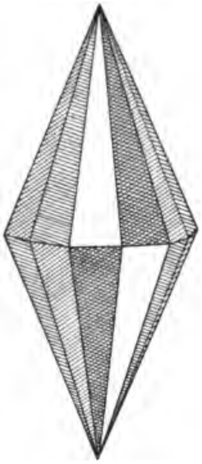
Fig. 242.

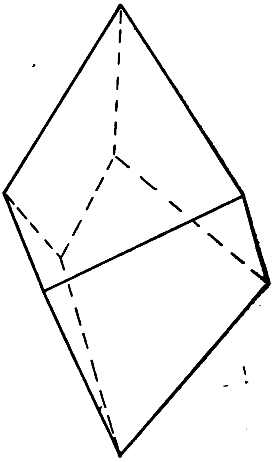
Fig. 243.

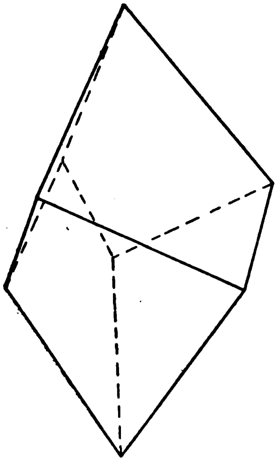
Fig. 244.

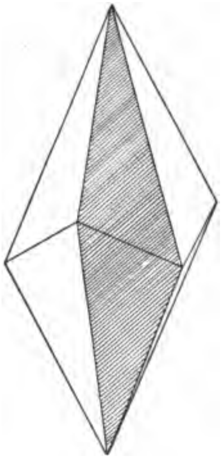
Fig. 245.

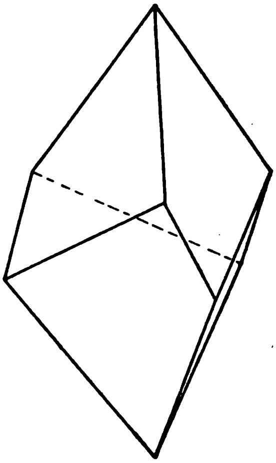
Fig. 246.

The symbols may be written as follows:

1) Positive right,

$$+r\left[\frac{na:pa:a:mc}{4}\right],\ +r\frac{mPn}{4},\quad \{hi\bar{k}l\},$$ **figure 243.**

2) Positive left,

$$+\, l\left[\frac{na : pa : a : mc}{4}\right],\ +\, l\,\frac{mPn}{4},\ \{k\,\bar{i}\,\bar{h}\,l\},\ \text{figure 241.}$$

3) Negative right,

$$-\, r\left[\frac{na : pa : a : mc}{4}\right],\ -\, r\,\frac{mPn}{4},\ \{i\,h\,\bar{k}\,l\},\ \text{figure 246.}$$

4) Negative left,

$$-\, l\left[\frac{na : pa : a : mc}{4}\right],\ -\, l\,\frac{mPn}{4},\ \{k\,\bar{h}\,\bar{i}\,l\},\ \text{figure 244.}$$

Forms 1 and 2, 3 and 4 are among themselves enantimorphous, while 1 and 3, 2 and 4 are congruent.

The polar axes of binary symmetry bisect the zigzag edges.

Trigonal prisms of the second order. Figure 247 shows that the hexagonal prism of this order yields two trigonal prisms. These are similar to those of the trigonal bipyramidal class, page 77.

The symbols are: $\pm\left[\frac{2a:2a:a:\infty c}{2}\right]$; $+\frac{\infty P2}{2}$, $\{h\,h\,\bar{2}\,\bar{h}\,o\}$; $-\frac{\infty P2}{2}$, $\{2\,h\,\bar{h}\,\bar{h}\,o\}$.

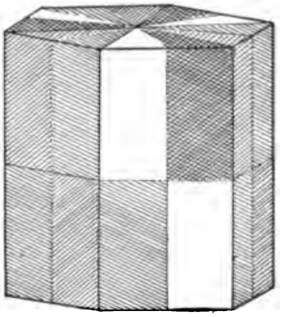

Fig. 247.

Ditrigonal Prisms. The dihexagonal prism now gives rise to two ditrigonal prisms as shown in figures 248, 249 and 250. These are similar, morphologically, to those of the ditrigonal bipyramidal class, page 59, but differ from them in respect to the position of the crystallographic axes. A comparison of figures 251 with 172 and 174 shows the difference.

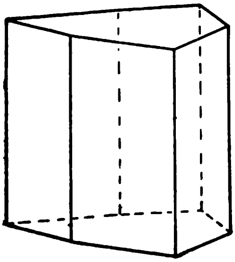

Fig. 248.

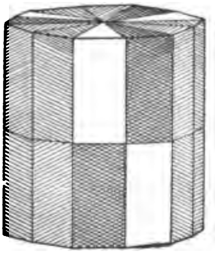

Fig. 249.

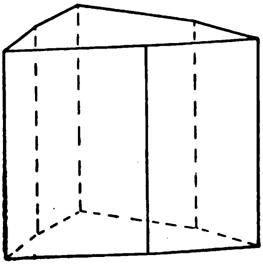

Fig. 250.

The symbols are: $\pm\left[\frac{na : pa : a : \infty c}{2}\right]$; $+\frac{\infty Pn}{2}$, $\{h\,i\,\bar{k}\,o\}$, figure 250; $-\frac{\infty Pn}{2}$, $\{k\,\bar{i}\,\bar{h}\,o\}$, figure 248.1)

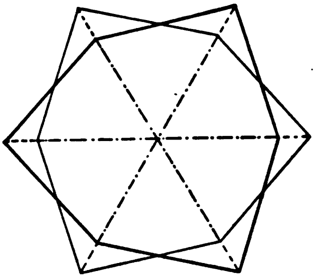

Fig. 251.

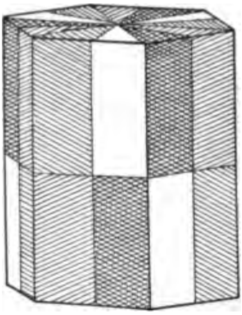

Fig. 252.

The other forms, the hexagonal prism of the first order and the basal pinacoid, remain unchanged, figure 252.

The important features of this class are given in the following table:

FORMS	SYMBOLS			Number of Faces	Solid Angles	
	Weiss	Naumann	Miller-Bravais		Trihedral	Tetrahedral
Rhombohedrons First order	$\pm\left[\frac{a : \infty a : a : mc}{2}\right]$	$\left\{\begin{matrix} +mR \\ -mR \end{matrix}\right.$	$\{h\,o\,\bar{h}\,l\}$ $\{o\,h\,\bar{h}\,l\}$	6	Apparently hemihedral	
Trigonal Bipyramids Second order	$\pm\left[\frac{2a : 2a : a : mc}{2}\right]$	$\left\{\begin{matrix} +\frac{mP2}{2} \\ -\frac{mP2}{2} \end{matrix}\right.$	$\{h\,h\,\overline{2h}\,l\}$ $\{2h\,\bar{h}\,\bar{h}\,l\}$	6	2	3
Trigonal Trapezohedrons	$\pm r$, $\pm l$ $\left[\frac{na : pa : a : mc}{4}\right]$	$\left\{\begin{matrix} +r\,\frac{mPn}{4} \\ +l\,\frac{mPn}{4} \\ -r\,\frac{mPn}{4} \\ -l\,\frac{mPn}{4} \end{matrix}\right.$	$\{h\,i\,\bar{k}\,l\}$ $\{k\,\bar{i}\,\bar{h}\,l\}$ $\{i\,h\,\bar{k}\,l\}$ $\{k\,\bar{h}\,\bar{i}\,l\}$	6	2 + 6	
Hexagonal Prism First order	$a : \infty a : a : \infty c$	∞P	$\{h\,o\,\bar{h}\,o\}$	6	Apparently holohedral	
Trigonal Prisms Second order	$\pm\left[\frac{2a : 2a : a : \infty c}{2}\right]$	$\left\{\begin{matrix} +\frac{\infty P2}{2} \\ -\frac{\infty P2}{2} \end{matrix}\right.$	$\{h\,h\,\overline{2h}\,o\}$ $\{2h\,\bar{h}\,\bar{h}\,o\}$	3		
Ditrigonal Prisms	$\pm\left[\frac{na : pa : a : \infty c}{2}\right]$	$\left\{\begin{matrix} +\frac{\infty Pn}{2} \\ -\frac{\infty Pn}{2} \end{matrix}\right.$	$\{h\,i\,\bar{k}\,o\}$ $\{k\,\bar{i}\,\bar{h}\,o\}$	6		
Basal Pinacoid	$\infty a : \infty a : \infty a : c$	OP	$\{0001\}$	2	Apparently holohedral	

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1) Often designated as *right* and *left* forms.

Combinations. Quartz, SiO_2, and Cinnabar, HgS, furnish excellent examples of minerals crystallizing in this class.

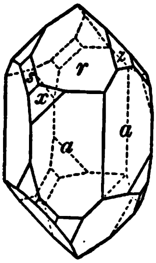

Fig. 253.

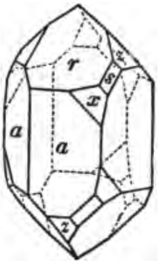

Fig. 254.

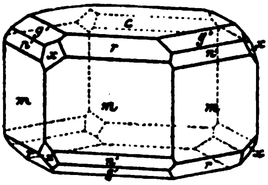

Fig. 255.

Figures 253 and 254. $a = \infty R\{10\bar{1}0\}$; $r = +R\{10\bar{1}1\}$; $z = -R\{0\bar{1}11\}$; s (figure 254) $= +\frac{2P2}{2}\{11\bar{2}1\}$, (figure 253) $-\frac{2P2}{2}\{2\bar{1}\bar{1}1\}$; x (figure 254) $= +r\frac{6P\frac{6}{5}}{4}\{51\bar{6}1\}$, (figure 253) $+l\frac{6P\frac{6}{5}}{4}\{6\bar{1}\bar{5}1\}$.

Figure 255. $c = 0R\{0001\}$; $m = \infty R\{10\bar{1}0\}$; $r = +R\{10\bar{1}1\}$; $g' = -\frac{1}{2}R\{01\bar{1}2\}$; $n' = -2R\{02\bar{2}1\}$; and $x = +l\frac{\frac{8}{5}P\frac{8}{5}}{4}\{83\bar{5}5.\}$ Cinnabar.

11. TRIGONAL RHOMBOHEDRAL CLASS.[1]

(*Rhombohedral Tetartohedrism.*)

Symmetry. This class possesses a principal axis of trigonal symmetry. A center of symmetry is also present, figure 256.

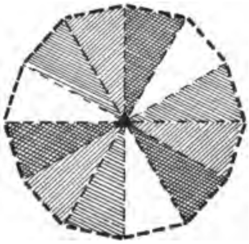

Fig. 256.

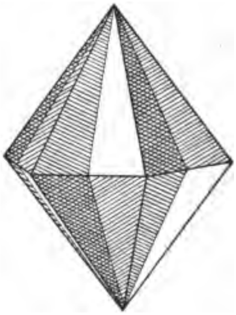

Fig. 257.

Rhombohedrons of the first order. The hexagonal bipyramid of the first order, figure 257, now yields two rhombohedrons which are identical, geometrically, with those of the ditrigonal scalenohedral class, page 60, figures 179 and 181.

---

[1] The *tri-rhombohedral group* of Dana.

The symbols are:

$$\pm\left[\frac{a : \infty a : a : mc}{2}\right];\ +m\mathrm{R}\ \{h\,0\,\bar{h}\,l\};\ -m\mathrm{R}\ \{0\,h\,\bar{h}\,l\}.$$

Rhombohedrons of the second order. Figure 259 shows the application of rhombohedral tetartohedrism on the hexagonal bipyramid of the second order. This form now yields the *rhombohedrons of the second order.*

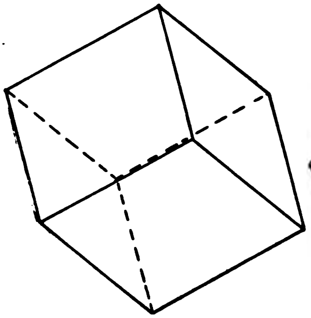
Fig. 258.

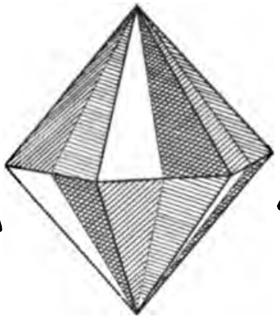
Fig. 259.

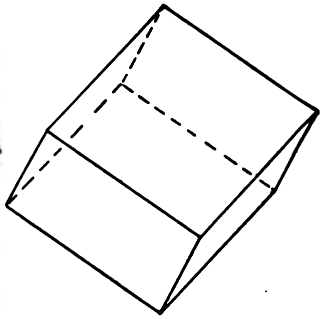
Fig. 260.

The symbols are:

$$\pm\left[\frac{2a : 2a : a : mc}{2}\right];\ +\frac{m\mathrm{P}2}{2}\ \{h\,h\,\bar{2}\,\bar{h}\,l\}\ \text{figure 260};\ -\frac{m\mathrm{P}2}{2}$$

$\{2\,h\,\bar{h}\,\bar{h}\,l\}$ figure 258.

Rhombohedrons of the third order. From the dihexagonal bipyramid four *rhombohedrons of the third order* result. Two of

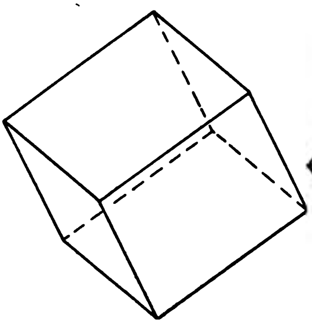
Fig. 261.

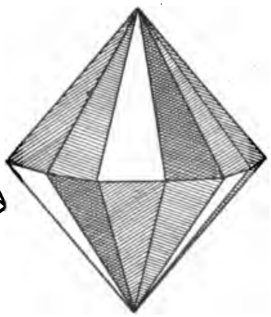
Fig. 262.

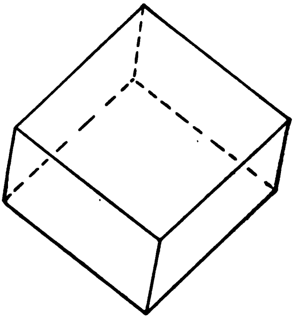
Fig. 263.

these are shown in figures 261, 262 and 263. The position of these rhombohedrons with respect to the crystallographic axes is shown by figures 236 and 237, which also indicate the relation existing between the rhombohedrons of the three orders.

The symbols are:

Positive right,

$$+r\left[\frac{na:pa:a:mc}{4}\right],\quad +r\frac{mPn}{4},\quad \{h\,i\,\bar{k}\,l\},\ \text{figure 263.}$$

Positive left,

$$+l\left[\frac{na:pa:a:mc}{4}\right],\quad +l\frac{mPn}{4},\quad \{k\,\bar{i}\,\bar{h}\,l\},\ \text{figure 261.}$$

Negative right,

$$-r\left[\frac{na:pa:a:mc}{4}\right],\quad -r\frac{mPn}{4},\quad \{i\,h\,\bar{k}\,l\}.$$

Negative left,

$$-l\left[\frac{na:pa:a:mc}{4}\right],\quad -l\frac{mPn}{4},\quad \{k\,\bar{h}\,\bar{i}\,l\}.$$

Hexagonal prisms of the third order. Figure 264 shows that the dihexagonal prism now yields two hexagonal prisms of the third order. Compare figures 205 and 207.

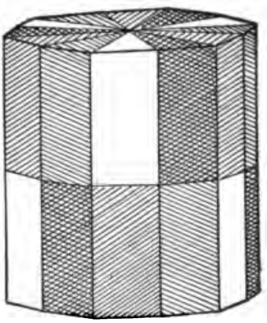

Fig. 264.

The symbols are:

$$\pm\left[\frac{na:pa:a:\infty c}{2}\right];\quad +\frac{\infty Pn}{2}\ \{h\,i\,\bar{k}\,o\};$$

$$-\frac{\infty Pn}{2}\ \{k\,\bar{i}\,\bar{h}\,o\}.$$

Other forms. The remaining holohedrons are unchanged by this tetartohedrism, see figures 265 and 266.

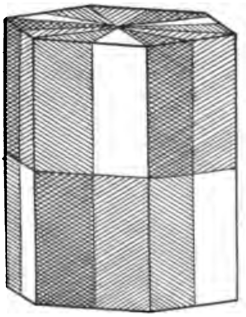

Fig. 265.

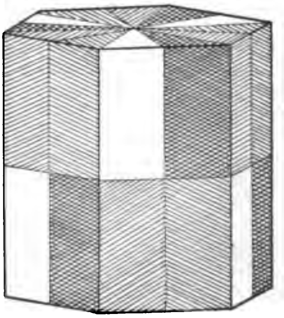

Fig. 266.

The principal features may be tabulated as follows:

FORMS	SYMBOLS			Faces	Solid Angles
	Weiss	Naumann	Miller-Bravais		Trihedral
Rhombo-hedrons First order	$\pm\left[\frac{a : \infty a : a : mc}{2}\right]$	$+mR$ $-mR$	$\{h\,o\,\bar{h}\,l\}$ $\{o\,h\,\bar{h}\,l\}$	6	Apparently of ditrigonal scalenohedral class
Rhombo-hedrons Second order	$\pm\left[\frac{2a : 2a : a : mc}{2}\right]$	$+\frac{mP2}{2}$ $-\frac{mP2}{2}$	$\{h\,h\,\overline{2h}\,l\}$ $\{2h\,\bar{h}\,\bar{h}\,l\}$	6	$2^{1)} + 6$
Rhombo-hedrons Third order	$\pm r, \pm l\left[\frac{na : pa : a : mc}{4}\right]$	$+r\,\frac{mPn}{4}$ $+l\,\frac{mPn}{4}$ $-r\,\frac{mPn}{4}$ $-l\,\frac{mPn}{4}$	$\{h\,i\,\bar{k}\,l\}$ $\{k\,\bar{i}\,\bar{h}\,l\}$ $\{i\,h\,\bar{k}\,l\}$ $\{k\,\bar{h}\,\bar{i}\,l\}$	6	$2^{1)} + 6$
Hexagonal Prism First order	$a : \infty a : a : \infty c$	∞P	$\{10\bar{1}0\}$	6	Apparently holohedral
Hexagonal Prism Second order	$2a : 2a : a : \infty c$	$\infty P2$	$\{11\bar{2}0\}$		
Hexagonal Prisms Third order	$\pm\left[\frac{na : pa : a : \infty c}{2}\right]$	$+\frac{\infty Pn}{2}$ $-\frac{\infty Pn}{2}$	$\{h\,i\,\bar{k}\,o\}$ $\{k\,\bar{i}\,\bar{h}\,o\}$	6	Apparently of hexagonal bipyramidal class
Basal Pinacoid	$\infty a ; \infty a : \infty a : c$	OP	$\{0001\}$	2	Apparently holohedral

1) Three equal edges.

Combinations. Dolomite, $CaMg(CO_3)_2$, and dioptase, H_2CuSiO_4, crystallize in this class.

Figure 267. $m = \infty P2, \{11\bar{2}0\}$; $r = -2R, \{02\bar{2}1\}$; $s = -l\,\frac{\frac{9}{4}P\frac{18}{17}}{4}$, $\{18.\bar{17}.\bar{1}.8\}$, dioptase.

Figure 268. $m = 4R, \{40\bar{4}1\}$; $r = R, \{10\bar{1}1\}$; $n = -\frac{\frac{16}{3}P2}{2}$, $\{16.\bar{8}.\bar{8}.3\}$ 1); $c = 0R, \{0001\}$, dolomite.

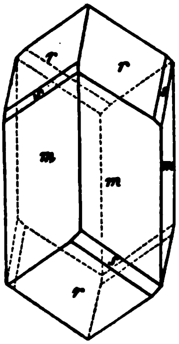

Fig. 267.

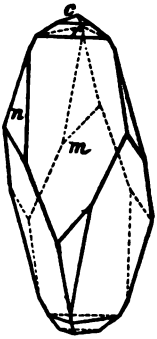

Fig. 268.

12. TRIGONAL PYRAMIDAL CLASS.

[*Ogdohedrism.*
Trigonal Tetartohedrism with Hemimorphism.]

Symmetry. The only element of symmetry is a polar axis of trigonal symmetry parallel to the *c* axis, figure 269.

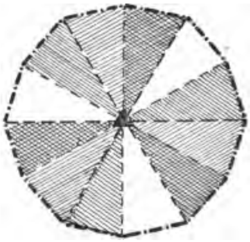

Fig. 269.

Forms. When hemimorphism becomes effective on forms of the trigonal bipyramidal class, page 76, it is evident the various bipyramids and the basal pinacoid give rise to upper and lower forms. The prisms, however, remain unchanged since they belong alike to the upper and lower poles.

The principal features of these forms are given in the following table:

---

1) Rhombohedron of the second order.

FORMS	SYMBOLS			FACES
	Weiss	Naumann	Miller-Bravais	
Trigonal Pyramids First order	$\pm\left[\frac{a : \infty a : a : mc}{4}\right] u, l$	$+\frac{mP}{4}u$ $+\frac{mP}{4}l$ $-\frac{mP}{4}u$ $-\frac{mP}{4}l$	$\{h0\bar{h}l\}$ $\{h0\bar{h}\bar{l}\}$ $\{0h\bar{h}l\}$ $\{0h\bar{h}\bar{l}\}$	3
Trigonal Pyramids Second order	$\pm\left[\frac{2a : 2a : a : mc}{4}\right] u,$	$+\frac{mP2}{4}u$ $+\frac{mP2}{4}l$ $-\frac{mP2}{4}u$ $-\frac{mP2}{4}l$	$\{hh\overline{2h}l\}$ $\{hh\overline{2h}\bar{l}\}$ $\{2h\bar{h}\bar{h}l\}$ $\{2h\bar{h}\bar{h}\bar{l}\}$	3
Trigonal Pyramids Third order	$\pm r, \pm l\left[\frac{na : pa : a : mc}{8}\right] u, l$	$+r\frac{mPn}{8}u$ $+l\frac{mPn}{8}u$ $-r\frac{mPn}{8}u$ $-l\frac{mPn}{8}u$ $+r\frac{mPn}{8}l$ $+l\frac{mPn}{8}l$ $-r\frac{mPn}{8}l$ $-l\frac{mPn}{8}l$	$\{hi\bar{k}l\}$ $\{k\bar{i}\bar{h}l\}$ $\{ih\bar{k}l\}$ $\{k\bar{h}\bar{i}l\}$ $\{ih\bar{k}\bar{l}\}$ $\{k\bar{h}\bar{i}\bar{l}\}$ $\{hi\bar{k}\bar{l}\}$ $\{k\bar{i}\bar{h}\bar{l}\}$	3
Trigonal Prisms First order	$\pm\left[\frac{a : \infty a : a : \infty c}{2}\right]$	$+\frac{\infty P}{2}$ $-\frac{\infty P}{2}$	$\{1010\}$ $\{01\bar{1}0\}$	Apparently of trigonal bipyramidal class
Trigonal Prisms Second order	$\pm\left[\frac{2a : 2a : a : \infty c}{2}\right]$	$+\frac{\infty P2}{2}$ $-\frac{\infty P2}{2}$	$\{11\bar{2}0\}$ $\{2\bar{1}\bar{1}0\}$	
Trigonal Prisms Third order	$\pm r, \pm l\left[\frac{na : pa : a : \infty c}{4}\right]$	$+r\frac{\infty Pn}{4}$ $+l\frac{\infty Pn}{4}$ $-r\frac{\infty Pn}{4}$ $-l\frac{\infty Pn}{4}$	$\{hi\bar{k}0\}$ $\{k\bar{i}\bar{h}0\}$ $\{ki\bar{k}0\}$ $\{k\bar{h}\bar{i}0\}$	
Basal Pinacoids	$\left[\frac{\infty a : \infty a : \infty a : c}{2}\right] u,$	$\frac{OP}{2}u$ $\frac{OP}{2}l$	$\{0001\}$ $\{000\bar{1}\}$	1

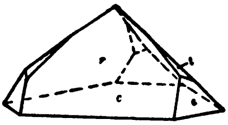

Fig. 270.

Combination. Figure 270. $r = + \frac{P}{4} u$, $\{10\bar{1}1\}$; $e = - \frac{2P}{4} u$, $\{02\bar{2}1\}$; $c = \frac{OP}{2} l$, $\{000\bar{1}\}$; $t = - r \frac{\frac{3}{4}P3}{8} u$, $\{21\bar{3}4\}$. Sodium periodate, $NaIO_4 + 3H_2O$.

TETRAGONAL SYSTEM.[1]

Crystallographic Axes. The tetragonal system includes all forms which can be referred to three perpendicular axes, two of which are equal and lie in a horizontal plane. These are termed the *secondary* or *lateral* axes and are designated as the *a* axes. Perpendicular to the plane of the lateral axes is the *principal* or *c* axis, which may be longer or shorter than the *a* axes. The axes, which bisect the angles between the *a* axes, are the *intermediate* axes. They are designated as the *b* axes in figure 271.

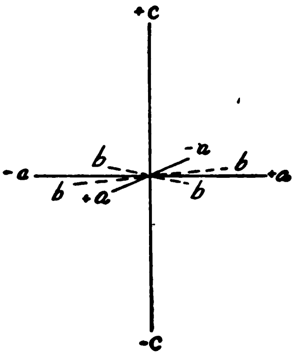

Fig. 271.

Crystals of this system are held so that the *c* axis is vertical and one of the *a* axes is directed towards the observer.

Since the lengths of the *a* and *c* axes differ, it is necessary to know the ratio existing between these axes, that is, the axial ratio, as was the case in the hexagonal system. Compare pages 7 and 43.

Classes of Symmetry. This system embraces seven classes of symmetry, as follows:

1.	Ditetragonal bipyramidal class	(*Holohedrism.*)
2.	Ditetragonal pyramidal class	*Holohedrism and hemimorphism.*
3.	Tetragonal scalenohedral class	(*Hemihedrism*).
4.	Tetragonal bipyramidal class	
5.	Tetragonal trapezohedral class	
6.	Tetragonal pyramidal class	*Hemihedrism and hemimorphism.*
7.	Tetragonal bisphenoidal class	(*Tetartohedrism*).

---

[1] Also termed *quadratic* or *pyramidal* system.

Classes 1, 3 and 4 are the most important. No representative of class 7 has yet been observed.

DITETRAGONAL BIPYRAMIDAL CLASS.[1]

(*Holohedrism.*)

Symmetry. a) **Planes.** In this class there are five planes of symmetry. The plane parallel to the plane of the secondary and intermediate axes is termed the principal plane. The vertical planes including the *c* and one of the *a* axes are called the *secondary* planes, while those which include one of the *b* axes are termed the *intermediate* planes, figure 272.

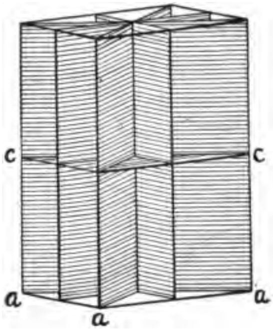

Fig. 272.

The principal and secondary planes divide space into eight equal parts, termed *octants*. The five planes, figure 272, divide it into sixteen equal sections. The five planes may be designated as follows:

1 Principal + 2 Secondary + 2 Intermediate = 5 Planes.

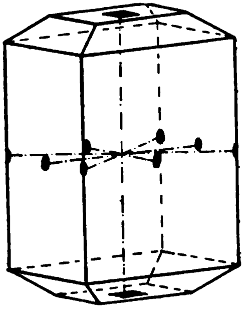

Fig. 273.

b) **Axes.** Parallel to the *c* axis, there is an axis of *tetragonal* symmetry. The axes parallel to the secondary and intermediate axes possess binary symmetry, figure 273. These may be written: 1 ■ + 2 ● + 2 ● = 5 axes.

c) **Center.** A center of symmetry is also present in this class. These elements of symmetry are shown in figure 274, which represents the projection of the most complex form upon the principal plane of symmetry.

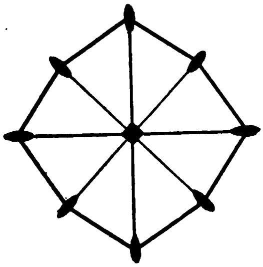

Fig. 274.

Tetragonal bipyramid of the first order. This form is analogous to the octahedron of the cubic system, page 19. But since

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[1] The *normal group* of Dana.

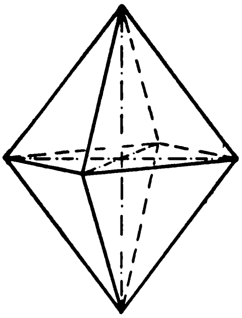

Fig 275.

the c axis differs from the secondary axes, the ratio must be written $(a : a : c)$, which would indicate the cutting of all three axes at unit distances,[1] figure 275. But since the intercept along the c axis may be longer or shorter than the unit length, the general ratio would read $(a : a : mc)$, where m is some value between zero and infinity. Like the octahedron, this form, the *tetragonal bipyramid*,[2] is bounded by eight faces which enclose space. The faces are equal isosceles triangles when the development is ideal.

The Naumann and Miller symbols are:

$$\mathrm{P}, \{111\}; \text{ or } m\mathrm{P}, \{h\,h\,l\}.$$

The principal crystallographic axis passes through the two tetrahedral angles of the same size, the secondary axes through the other four equal tetrahedral angles, while the intermediate axes bisect the horizontal edges.

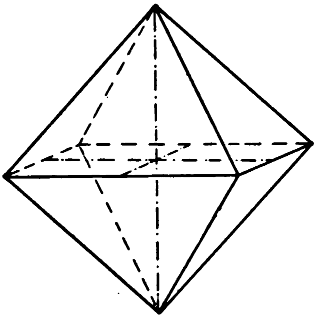

Fig. 276.

Tetragonal bipyramid of the second order. The faces of this form cut the c and one of the a axes, but extend parallel to the other. The parametral ratio is, therefore, $a : \infty a : mc$. This requires eight faces to enclose space and the form is the *bipyramid of the second order*, figure 276. The symbols according to Naumann and Miller are: $m\mathrm{P}\infty$, $\{o\,h\,l\}$.

In form this bipyramid does not differ from the preceding, but they can be readily distinguished on account of their position in respect to the secondary axes. In this form, the secondary axes bisect the horizontal edges and the intermediate axes pass through the four equal tetrahedral angles. This is the opposite of what was noted with the bipyramid of the first order, compare figures 275 and 276. Hence, the bipyramid of the first order is always held so that an edge is directed toward the observer, whereas the bipyramid of the second order presents a face. In both bipyramids the principal axis passes through the two equal tetrahedral angles.

---

[1] Indicating a *unit* form, compare page 7.

[2] The more the ratio $a : c$ approaches 1 : 1, the more does this form simulate the octahedron. This tendency of forms to simulate those of a higher grade of symmetry is spoken of as *pseudosymmetry*.

Ditetragonal bipyramid. The faces of this bipyramid cut the two secondary axes at different distances, whereas the intercept along the c axis may be unity or mc. Sixteen such faces are possible and, hence, the term *ditetragonal bipyramid* is used, figure 277.1)

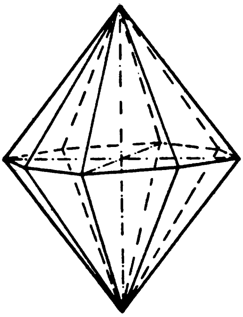

Fig. 277.

The symbols are:

$$(a : na : mc), \quad m\mathrm{P}n, \quad \{hkl\}.$$

Since the polar edges2) are alternately dissimilar it follows that the faces are equal, similar scalene triangles. The ditetragonal bipyramid possessing equal polar edges is crystallographically an impossible form, for then the ratio $a : na : mc$ would necessitate a value for n equal to the tangent of 67° 30′, namely, the irrational value 2.4142+.3)

From the above it follows that when n is less than 2.4142+ the ditetragonal bipyramid simulates the tetragonal bipyramid of the first order, and finally when it equals 1, it passes over into that form. On the other hand, if n is greater than 2.4142+ it approaches more the bipyramid of the second order, and when it is equal to infinity passes over into that form. Hence, $n > 1 < \infty$. Figure 278 illustrates this clearly. It is also to be noted, that when n is less than 2.4142+ the secondary axes pass through the more acute angles, whereas, when n is greater than 2.4142+ they join the more obtuse. In figure 278, outline 1 represents the cross-section of the tetragonal bipyramid of the first order, 2 that

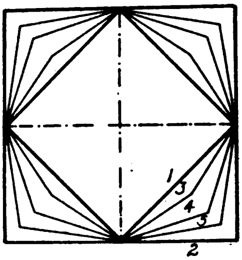

Fig. 278.

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1) Compare figure 24, page 10.

2) Compare footnote, page 48.

3) See also page 49.

of the second order, and 3, 4 and 5 ditetragonal bipyramids where *n* equals $\frac{3}{2}$, 3, and 6, respectively.

Tetragonal prism of the first order. If the value of the intercept along the *c* axis in the tetragonal bipyramid of this order becomes infinity, the number of the faces of the bipyramid is reduced to four giving rise to the *tetragonal prism of the first order*, figure 279. This is an open form and possesses the following symbols:

$$(a : a : \infty c), \quad \infty P, \quad \{110\}.$$

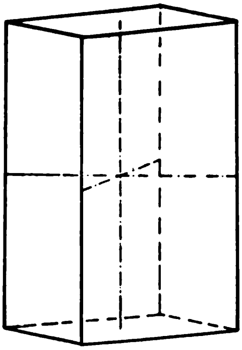

Fig. 279.

The secondary axes join opposite edges, hence, an edge is directed toward the observer.

Tetragonal prism of the second order. The same relationship exists between this form and its corresponding bipyramid as was noted on the preceding form.

The symbols are:

$$(a : \infty a : \infty c), \quad \infty P \infty, \quad \{100\}.$$

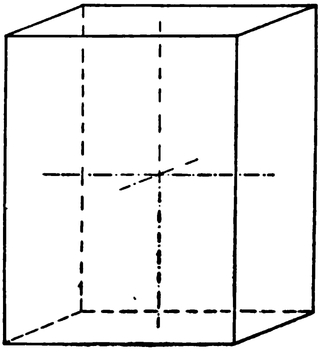

Fig. 280.

This is also an open form consisting of four faces, figure 280. The secondary axes join the centers of opposite faces. Hence, a face is directed toward the observer.

Ditetragonal prism. As is obvious, this form consists of eight faces possessing the following symbols:

$$a : na : \infty c, \quad \infty Pn, \quad \{hko\}.$$

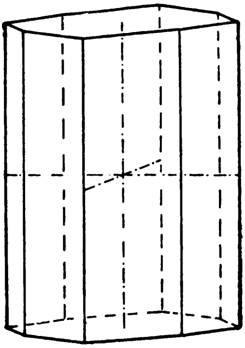

Fig. 281.

What was said on page 94 concerning the polar angles and the position of the secondary axes applies here also. Figure 281 represents a ditetragonal prism.

Basal Pinacoid. This form is similar to that of the hexagonal system, page 51. It is parallel to the secondary axes but cuts the *c* axis. The symbols may be written:

(∞*a* : ∞*a* : *c*), OP, {001}.

This form consists of but two faces. They are shown in combination with the three prisms in figures 279, 280, and 281.

These are the seven simple forms possible in the tetragonal system. The following table shows the chief characteristics of each.

FORMS	SYMBOLS			Faces	Solid Angles	
	Weiss	Naumann	Miller		Tetrahedral	Octahedral
Unit Bipyramid First order	*a* : *a* : *c*	*P*	{*111*}	8	2 + 4	—
Modified Bipyramids First order	*a* : *a* : *mc*	*mP*	{*hhl*}	8	2 + 4	—
Bipyramids Second order	*a* : ∞*a* : *mc*	*mP*∞	{*h o l*}	8	2 + 4	—
Ditetragonal Bipyramids	*a* : *na* : *mc*	*mPn*	{*h k l*}	16	4 + 4	2
Prism First order	*a* : *a* : ∞*c*	∞*P*	{*110*}	4	—	—
Prism Second order	*a* : ∞*a* : ∞*c*	∞*P*∞	{*100*}	4	—	—
Ditetragonal Prisms	*a* : *na* : ∞*c*	∞*Pn*	{*h k o*}	8	—	—
Basal Pinacoid	∞*a* : ∞*a* : *c*	*OP*	{*001*}	2	—	—

Relationship of forms. This is clearly expressed by the following diagram. Compare pages 23 and 52.

$$\begin{tikzcd}
& & \infty a : \infty a : c \arrow[dl, dash] \arrow[dr, dash] & & \\
& a : \infty a : mc \arrow[ddl, dash] \arrow[dr, dash] & & a : a : mc \arrow[dl, dash] \arrow[ddr, dash] & \\
& & a : na : mc \arrow[d, dash] & & \\
a : \infty a : \infty c \arrow[rr, dash] & & a : na : \infty c \arrow[rr, dash] & & a : a : \infty c
\end{tikzcd}$$

Combinations. Some of the more common combinations are illustrated by the following figures:

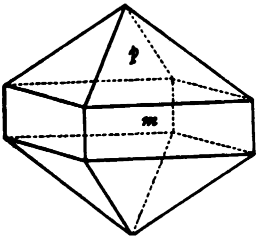

Fig. 282.

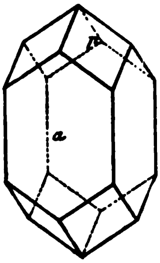

Fig. 283.

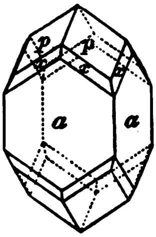

Fig. 284.

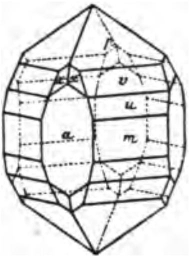

Fig. 285.

Figures 282 to 286. $m = \infty P\{110\}$, $p = P\{111\}$, $a = \infty P \infty \{100\}$, $u = 3P\{331\}$, $v = 2P\{221\}$, and $x = 3P3\{311\}$. These combinations are to be observed on zircon, $ZrSiO_4$.

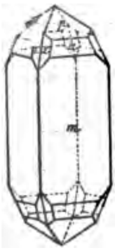

Fig. 286.

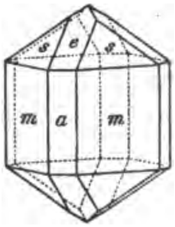

Fig. 287.

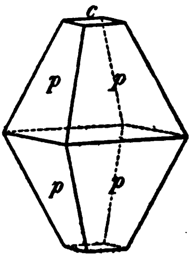

Fig. 288.

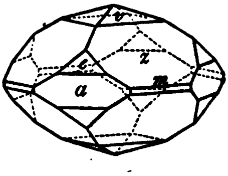

Fig. 289.

Figure 287. $m_{,} = \infty P\{110\}$, $a = \infty P\infty\{100\}$, $e = P\infty\{101\}$, and $s = P\{111\}$. Cassiterite, Sn_2O_4.

Figures 288 and 289. $p = P\{111\}$, $c = 0P\{001\}$, $z = \frac{1}{3}P\{113\}$, $v = \frac{1}{7}P\{117\}$, $m = \infty P\{110\}$, $a = \infty P\infty\{100\}$, and $e = P\infty\{101\}$. These combinations occur on Anatase, TiO_2.

2. DITETRAGONAL PYRAMIDAL CLASS.[1]

(*Holohedrism with Hemimorphism.*)

Symmetry. Since hemimorphism is effective on forms of this class, the principal plane and the four axes of binary symmetry disappear. The remaining elements are, therefore, two secondary and two intermediate planes and a polar axis of tetragonal symmetry, figure 290.

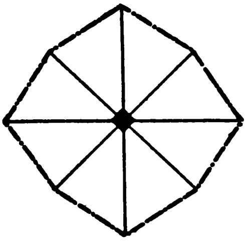

Fig. 290.

Forms. The forms of this class are analogous to those of the dihexagonal pyramidal class, page 53, and the ditetragonal bipyramids and tetragonal bipyramids of the first and second orders as well as the basal pinacoid are now to be considered as divided into *upper* and *lower* forms. For example, the ditetragonal bipyramid now yields an *upper* and a *lower* ditetragonal *pyramid*, each consisting of eight faces. The three prisms, however, remain unaltered; they are apparently holohedral.

---

[1] The *hemimorphic group* of Dana.

The principal features are summarized in the following table:

FORMS	SYMBOLS			Faces	Solid Angles	
	Weiss	Naumann	Miller		Tetrahedral	Octahedral
Upper and Lower Pyramids First order	$\left[\frac{a : a : mc}{2}\right]\frac{u}{l}$	$\frac{mP}{2}u$ $\frac{mP}{2}l$	$\{111\}$ $\{11\bar{1}\}$	4	1	—
Upper and Lower Pyramids Second order	$\left[\frac{a : \infty a : mc}{2}\right]\frac{u}{l}$	$\frac{mP\infty}{2}u$ $\frac{mP\infty}{2}l$	$\{h\,o\,l\}$ $\{h\,o\,\bar{l}\}$	4	1	—
Upper and Lower Ditetragonal Pyramids	$\left[\frac{a : na : mc}{2}\right]\frac{u}{l}$	$\frac{mPn}{2}u$ $\frac{mPn}{2}l$	$\{h\,k\,l\}$ $\{h\,k\,\bar{l}\}$	8	—	1
Tetragonal Prism First order	$a : a : \infty c$	∞P	$\{110\}$	4	Apparently holohedral	
Tetragonal Prism Second order	$a : \infty a : \infty c$	$\infty P\infty$	$\{100\}$	4		
Ditetragonal Prisms	$a : na : \infty c$	∞Pn	$\{h\,k\,o\}$	8		
Upper and Lower Basal Pinacoids	$\left[\frac{\infty a : \infty a : c}{2}\right]\frac{u}{l}$	$\frac{OP}{2}u$ $\frac{OP}{2}l$	$\{100\}$ $\{00\bar{1}\}$	1	—	—

Combinations. Figure 291. $x = \frac{\frac{1}{3}P}{2}\,u$, $\{113\}$; $o = \frac{P}{2}\,u$, $\{111\}$; $w = \frac{P}{2}\,l$, $\{11\bar{1}\}$ Silver fluoride, $AgF.H_2O$.

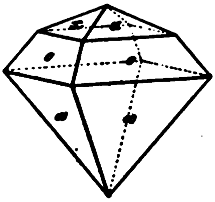

Fig. 291.

Figure 292. $a = \infty P \infty$ $\{100\}$; $o = \frac{P}{2}\,u$, $\{111\}$; $c = \frac{OP}{2}\,u$, $\{001\}$; $w = \frac{P}{2}\,l$, $\{11\bar{1}\}$. Observed on penta-erythrite, $C_5H_{12}O_4$.

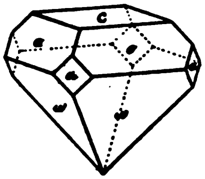

Fig. 292.

TETRAGONAL HEMIHEDRISMS.

In this system three types of hemihedrisms are possible, figures 293, 294 and 295.

a) **Sphenoidal hemihedrism.** The principal and two secondary planes divide space into octants. All faces in alternate octants are suppressed, the others expanded, figure 293.

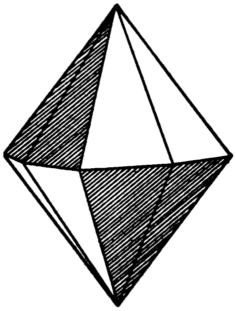

Fig. 293.

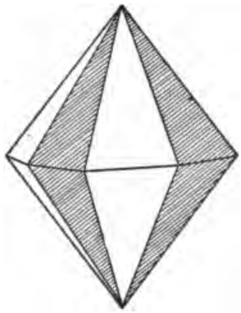

Fig. 294.

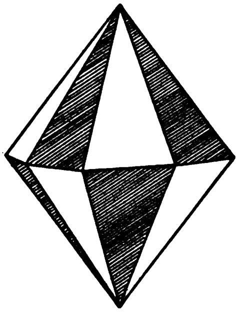

Fig. 295.

b) **Pyramidal Hemihedrism.** The two secondary and the two intermediate planes of symmetry divide space into eight equal sections, figure 272. All faces lying wholly within alternate sections are extended, the others suppressed, figure 294.

c) **Trapezohedral hemihedrism.** By means of the five planes of symmetry sixteen equal sections result. In this type of hemihedrism all faces lying wholly within such sections are alternately extended and suppressed, figure 295.

The pyramidal and sphenoidal types of hemihedrism are the most important.

3. TETRAGONAL SCALENOHEDRAL CLASS.[1]

(*Sphenoidal Hemihedrism.*)

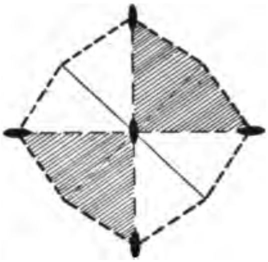

Fig. 296.

Symmetry. In this class the elements of symmetry consist of two intermediate planes of symmetry, two axes of binary symmetry parallel to the secondary crystallographic axes, and one binary axis parallel to the principal or *c* axis, figure 296. The sphenoidal hemihedrism is analogous to the tetrahedral and rhombohedral hemihedrisms of the cubic and hexagonal systems, respectively; see pages 26 and 60.

Tetragonal bisphenoids. Figure 298 shows the application of the sphenoidal hemihedrism to the bipyramid of the first order. It is obvious from this figure that the bipyramid of the first order now yields two new correlated forms, each bounded by four faces. When the development is ideal, these faces are equal isosceles triangles. On account of the resulting forms resembling a wedge, they are termed the *positive* and *negative tetragonal bisphenoids of the first order.*

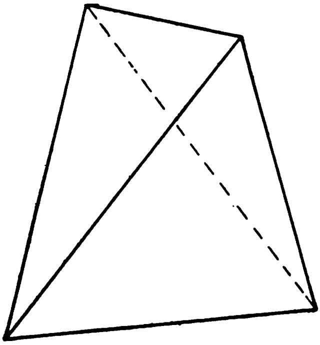

Fig. 297.

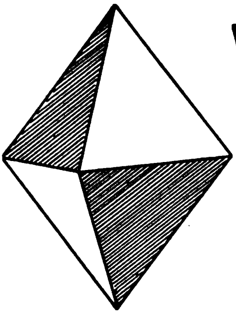

Fig. 298.

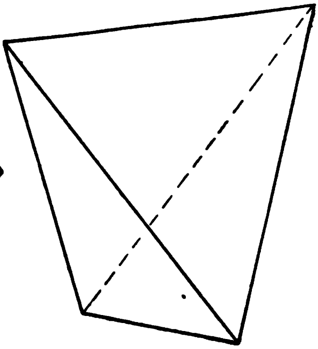

Fig. 299.

The symbols are:

$$\pm\left[\frac{a : a : mc}{2}\right];\ +\frac{mP}{2},\ \kappa\{h\,h\,l\}\ \text{figure 299;}$$

$$-\frac{mP}{2},\ \kappa\{h\,\bar{h}\,l\}\ \text{figure 297.}$$

---

[1] *Sphenoidal group* of Dana.

The secondary crystallographic axes bisect the four edges of equal length, while the principal axis passes through the centers of the other two.

Tetragonal scalenohedrons. It is readily to be seen from figure 301, that the ditetragonal bipyramid also yields two new correlated forms. These consist of eight similar scalene triangles, and are termed the *positive* and *negative tetragonal scalenohedrons*.

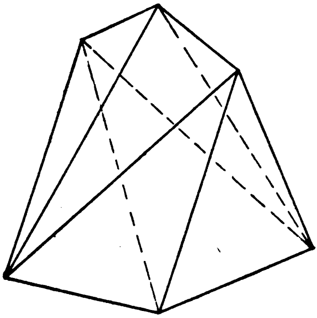

Fig. 300.

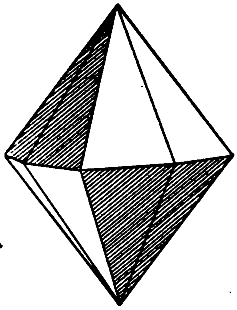

Fig. 301.

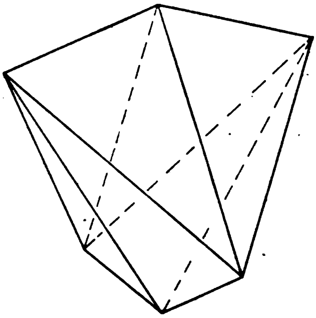

Fig. 302.

The symbols are:

$$\pm\left[\frac{a : na : mc}{2}\right];\ +\frac{mPn}{2},\ \kappa\,\{h\,k\,l\}\ \text{figure 302};$$

$$-\frac{mPn}{2},\ \kappa\,\{h\,\bar{k}\,l\}\ \text{figure 300}.$$

The principal axis joins the tetrahedral angles, which possess two pairs of equal edges. The secondary axes bisect the four zigzag edges.

The sphenoids and scalenohedrons are congruent and, hence, the positive and negative may be made to coincide by means of a revolution through 90° about the principal axis.

The other tetragonal forms are apparently holohedral as a study of figures 296, 298, and 301 will show.

The characteristics of the forms of the tetragonal scalenohedral class may be tabulated as follows:

FORMS	SYMBOLS			Faces	Solid Angles	
	Weiss	Naumann	Miller		Trihedral	Tetrahedral
Bisphenoids First order	$\pm\left[\frac{a : a : mc}{2}\right]$	$+\frac{mP}{2}$ $-\frac{mP}{2}$	$\kappa\{hhl\}$ $\kappa\{h\bar{h}l\}$	4	4	—
Bipyramid Second order	$a : \infty a : mc$	$mP\infty$	$\kappa\{h0l\}$	Apparently holohedral		
Tetragonal Scalenohedrons	$\pm\left[\frac{a : na : mc}{2}\right]$	$+\frac{mPn}{2}$ $-\frac{mPn}{2}$	$\kappa\{hkl\}$ $\kappa\{h\bar{k}l\}$	8	—	2+4
Prism First order	$a : a : \infty c$	∞P	$\kappa\{110\}$	Apparently holohedral		
Prism Second order	$a : \infty a : \infty c$	$\infty P\infty$	$\kappa\{100\}$			
Ditetragonal Prisms	$a : na : \infty c$	∞Pn	$\kappa\{hk0\}$			
Basal Pinacoid	$\infty a : \infty a : c$	OP	$\kappa\{001\}$			

Combinations. Figure 303, $m = \infty P$, $\{110\}$; $o = +\frac{P}{2}$, $\{111\}$; and $c = OP$, $\{001\}$. Urea, CH_4N_2O.

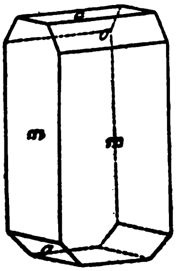

Fig. 303.

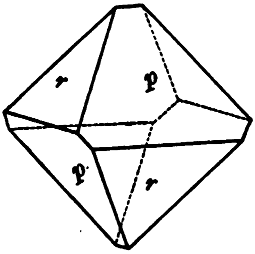

Fig. 304.

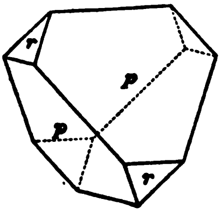

Fig. 305.

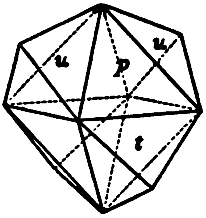

Fig. 306.

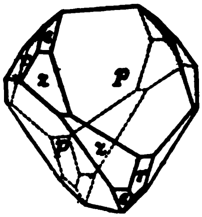

Fig. 307.

Figures 304 to 307. $p = +\frac{P}{2}$, $\{111\}$; $r = -\frac{P}{2}$, $\{1\bar{1}1\}$; $e =$ P∞, $\{101\}$; $z =$ 2P∞, $\{201\}$; $c =$ OP, $\{001\}$; $m = \infty$P, $\{110\}$; $t = -\frac{2P}{2}$, $\{22\bar{1}\}$; $u =$ P2, $\{212\}$. These combinations occur on chalcopyrite, $CuFeS_2$.

4. TETRAGONAL BIPYRAMIDAL CLASS.[1]

(*Pyramidal Hemihedrism.*)

Symmetry. The elements of this class are one principal plane, a center, and an axis of tetragonal symmetry, as shown in figure 312.

Tetragonal bipyramids of the third order. From figure 309 it is obvious that by means of the pyramidal hemihedrism, the dihexagonal bipyramid yields two new forms, each bounded by eight equal isosceles triangles, figures 308 and 310, termed the *tetragonal bipyramids of the third order.*

The symbols are:

$$\pm\left\{\frac{a : na : mc}{2}\right\};\quad +\left[\frac{mPn}{2}\right],\ \pi\{hkl\},\ \text{figure 310;}$$

$$-\left[\frac{mPn}{2}\right],\ \pi\{h\bar{k}l\},\ \text{figure 308.}$$

In form these bipyramids do not differ from those of the first and second orders. Figures 311 and 313 show the positions occupied by the bipyramids of the three orders and it is obvious that the position of the forms of the *third* order is intermediate between that of the other two.

Tetragonal prisms of the third order. In an analogous manner the ditetragonal prism, figure 315, yields two correlated *prisms of the third order*, each consisting of four faces. The position of these prisms in respect to the crystallographic axes is illustrated by figures 311 and 313.

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1) Dana's *pyramidal group.*

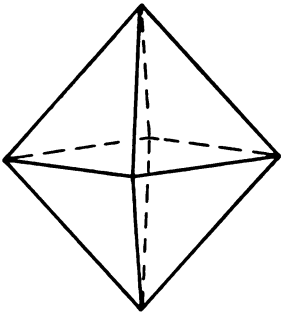
Fig. 308.

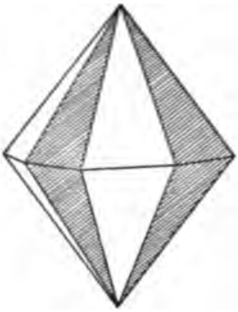
Fig. 309.

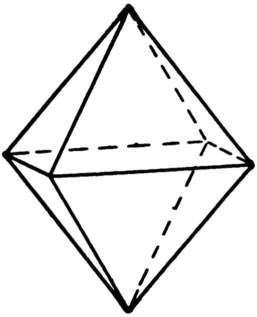
Fig. 310.

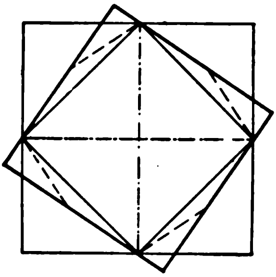
Fig. 311.

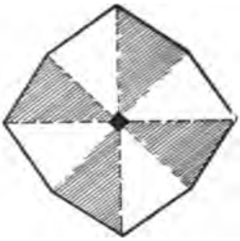
Fig. 312.

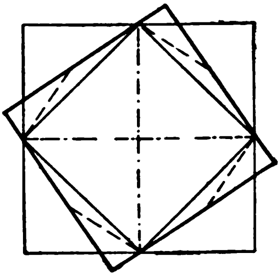
Fig. 313.

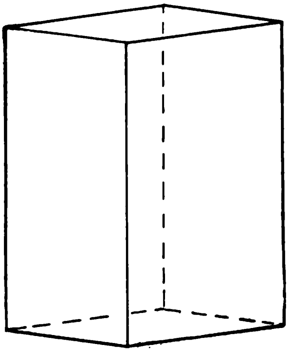
Fig. 314.

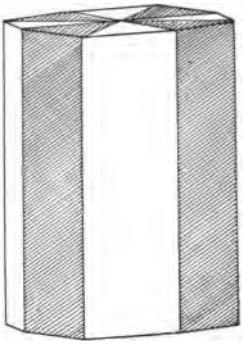
Fig. 315.

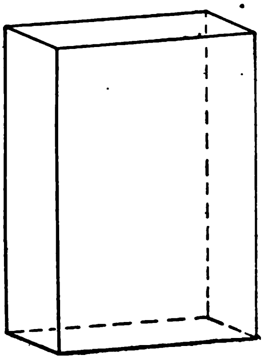
Fig. 316.

The symbols are:

$\pm\left[\frac{a : na : \infty c}{2}\right]$; $+\left[\frac{\infty P n}{2}\right]$, $\pi\{hk0\}$, figure 316.

$-\left[\frac{\infty P n}{2}\right]$, $\pi\{h\bar{k}0\}$, figure 314.

Being open forms, they are shown in combination with the basal pinacoid.

Other forms. The other holohedrons, bipyramids and prisms of the first and second orders as also the basal pinacoid, remain unaltered by this type of hemihedrism because their faces do not lie wholly within the sections formed by the secondary and intermediate planes of symmetry, compare figures 309, 312, and 315. They are, hence, only apparently holohedral.

The chief features of the forms of the tetragonal bipyramidal class are given in the following table:

FORMS	SYMBOLS			Faces	Solid Angles
	Weiss	Naumann	Miller		Tetrahedral
Bipyramids First order	$a : a : mc$	mP	$\pi\{hhl\}$	Apparently holohedral	
Bipyramids Second order	$a : \infty a : mc$	$mP\infty$	$\pi\{h0l\}$		
Bipyramids Third order	$\pm\left\{\frac{a : na : mc}{2}\right\}$	$+\left[\frac{mPn}{2}\right]$ $-\left[\frac{mPn}{2}\right]$	$\pi\{hkl\}$ $\pi\{h\bar{k}l\}$	8	2 + 4
Prism First order	$a : a : \infty c$	∞P	$\pi\{110\}$	Apparently holohedral	
Prism Second order	$a : \infty a : \infty c$	$\infty P\infty$	$\pi\{100\}$		
Prisms Third order	$\pm\left\{\frac{a : na : \infty c}{2}\right\}$	$+\left[\frac{\infty Pn}{2}\right]$ $-\left[\frac{\infty Pn}{2}\right]$	$\pi\{hk0\}$ $\pi\{h\bar{k}0\}$	4	—
Basal Pinacoid	$\infty a : \infty a : c$	$0P$	$\pi\{001\}$	Apparently holohedral	

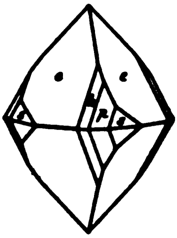

Fig. 317.

Combination. Figure 317. $e = P\infty$, $\pi\{101\}$; $p = P$, $\pi\{111\}$; $h = +\left[\frac{P3}{2}\right]$, $\pi\{313\}$; $s = -\left[\frac{3P3}{2}\right]$, $\pi\{1\bar{3}1\}$. This combination has been observed on the mineral scheelite, $CaWO_4$.

5. TETRAGONAL TRAPEZOHEDRAL CLASS.[1]

(Trapezohedral hemihedrism.)

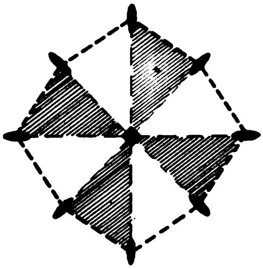

Fig. 318.

Symmetry. This class possesses a principal axis of tetragonal symmetry and four axes of binary symmetry. Figure 318 shows these elements as well as the application of the trapezohedral hemihedrism.

Tetragonal trapezohedrons. The only holohedron which can be affected by trapezohedral hemihedrism, page 101, is the ditetragonal bipyramid, figure 320. This bipyramid yields two new correlated forms, bounded by eight faces, which are similar trapeziums, figures 321 and 319. Being *enantiomorphous*, they are termed the *right* and *left tetragonal trapezohedrons.*

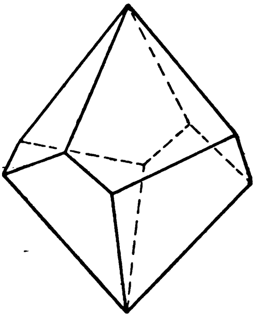

Fig. 319.

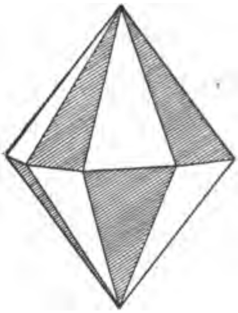

Fig. 320.

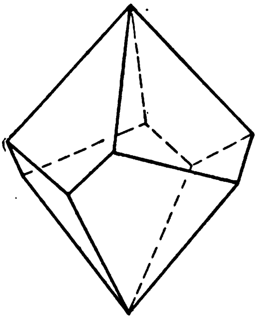

Fig. 321.

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[1] Dana calls this the *trapezohedral group.*

The symbols are:

$$r,\ l\left\{\frac{a : na : mc}{2}\right\};\quad r\,\frac{mPn}{2},\ \tau\{hkl\},\ \text{figure 321};$$

$$l\,\frac{mPn}{2},\ \tau\{h\bar{k}l\},\ \text{figure 319}.$$

All other forms are apparently holohedral because their faces do not lie wholly within each of the sixteen sections resulting from the intersection of the five planes of symmetry.

The chief features of the forms of this class are given below:

FORMS	SYMBOLS			Faces	Solid Angles	
	Weiss	Naumann	Miller		Trihedral	Tetrahedral
Bipyramids First order	$a : a : mc$	mP	$\tau\{hhl\}$	Apparently holohedral		
Bipyramids Second order	$a : \infty a : mc$	$mP\infty$	$\tau\{h0l\}$			
Tetragonal Trapezohedrons	$r,\ l\left\{\frac{a : na : mc}{2}\right\}$	$r\,\frac{mPn}{2}$ $l\,\frac{mPn}{2}$	$\tau\{hkl\}$ $\tau\{h\bar{k}l\}$	8	8	2
Prism First order	$a : a : \infty c$	∞P	$\tau\{110\}$	Apparently holohedral		
Prism Second order	$a : \infty a : \infty c$	$\infty P \infty$	$\tau\{100\}$			
Ditetragonal Prism	$a : na : \infty c$	∞Pn	$\tau\{hk0\}$			
Basal Pinacoid	$\infty a : \infty a : c$	$0P$	$\tau\{001\}$			

Thus far, the tetragonal trapezohedron has not been observed on either minerals or artificial salts. There are, however, a number of compounds like sulphate of strychnine, $(C_{21}H_{22}N_2O_2)_2H_2SO_4+6H_2O$, and sulphate of nickel, $NiSO_4+6H_2O$, which have been referred to this class of symmetry by means of etch figures.

6. TETRAGONAL PYRAMIDAL CLASS.[1]

(*Pyramidal Hemihedrism with Hemimorphism.*)

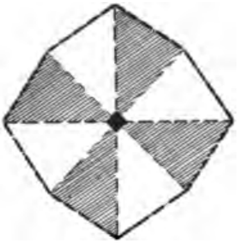

Fig. 322.

Symmetry. If the forms of the tetragonal bipyramidal class, page 150, become hemimorphic, the principal plane and center of symmetry disappear. Hence, the only element of symmetry of this class is a polar axis of tetragonal symmetry which is parallel to the *c* axis, figure 322.

Forms. The bipyramids of the tetragonal bipyramidal class now yield *upper* and *lower* pyramids. The basal pinacoid is also divided into two forms. The prisms remain unaltered.

Since the ditetragonal bipyramid is now divided into four pyramids, this class is sometimes considered a tetartohedral class.

The chief features of the forms are given in the following table:

FORMS	SYMBOLS			FACES
	Weiss	Naumann	Miller	
Pyramids First order	$\left[\frac{a:a:mc}{2}\right]\frac{u}{l}$	$\frac{mP}{2}u$ $\frac{mP}{2}l$	$\{hhl\}$ $\{hh\bar{l}\}$	4
Pyramids Second order	$\left[\frac{a:\infty a:mc}{2}\right]\frac{u}{l}$	$\frac{mP\infty}{2}u$ $\frac{mP\infty}{2}l$	$\{h0l\}$ $\{h0\bar{l}\}$	4
Pyramids Third order	$\pm\left[\frac{a:na:mc}{4}\right]\frac{u}{l}$	$+\left[\frac{mPn}{4}\right]u$ $+\left[\frac{mPn}{4}\right]l$ $-\left[\frac{mPn}{4}\right]u$ $-\left[\frac{mPn}{4}\right]l$	$\{hkl\}$ $\{hk\bar{l}\}$ $\{h\bar{k}l\}$ $\{h\bar{k}\bar{l}\}$	4
Prism First order	$a:a:\infty c$	Apparently of tetragonal bipyramidal class		
Prism Second order	$a:\infty a:\infty c$			
Prisms Third order	$\pm\left[\frac{a:na:\infty c}{2}\right]$			
Basal Pinacoids	$\left[\frac{\infty a:\infty a:c}{2}\right]\frac{u}{l}$	$\frac{OP}{2}u$ $\frac{OP}{2}l$	$\{001\}$ $\{00\bar{1}\}$	1

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[1] The *pyramidal-hemimorphic group* of Dana.

Combination. The mineral wulfenite, $PbMoO_4$, crystallizes in this class, figure 323. $o = \frac{P}{2}\ u,\ \{111\};\ o' = \frac{P}{2}\ l,\ \{11\bar{1}\};\ p = +\left[\frac{\infty P\frac{4}{3}}{2}\right],\ \{430\};\ c = \frac{OP}{2}\ l,\ \{00\bar{1}\}.$

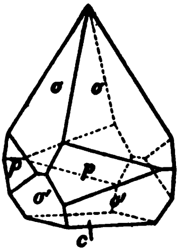

Fig. 323.

7. *TETRAGONAL BISPHENOIDAL CLASS.*[1]

(*Tetartohedrism.*)

Symmetry. The only element of symmetry remaining in this class is an axis of binary symmetry parallel to the *c* axis, figure 324.

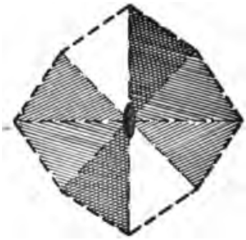

Fig. 324.

Tetartohedrism. The forms of this class may be conceived as derived from the holohedrons by the simultaneous occurrence of ~~either the sphenoidal and trapezohedral, or~~ the sphenoidal and pyramidal hemihedrisms; compare figures 293, 294, and 295. The simultaneous occurrence, however, of the pyramidal and trapezohedral hemihedrisms give rise to the forms as described in the preceding class.

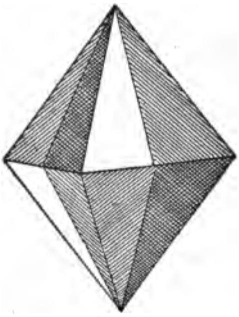

Fig. 325.

Bisphenoids of the first order. Figure 325 shows that the bipyramid of the first order now yields two bisphenoids of the same order. Compare figure 298.

The symbols are:

$$\pm\left[\frac{a:a:mc}{2}\right];\ +\frac{mP}{2}\ \{h\,h\,l\};\ -\frac{mP}{2}\ \{h\,\bar{h}\,l\}.$$

Bisphenoids of the second order. The bipyramid of the second order, figure 327, gives rise to the *positive* and *negative bisphenoids*

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[1] Dana terms this the *tetartohedral group*.

of the second order. These bisphenoids bear the same relation to those of the first order as do the corresponding bipyramids.

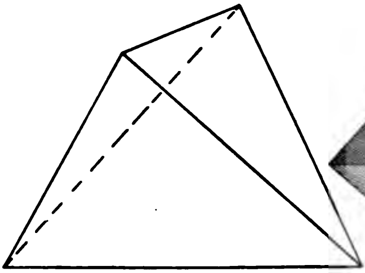

Fig. 326.

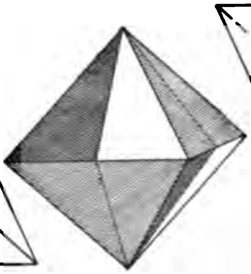

Fig. 327.

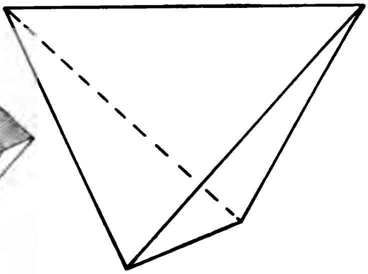

Fig. 328.

The symbols are:

$\pm\left[\frac{a : \infty a : mc}{2}\right]$; $+\frac{m\mathrm{P}\infty}{2}$ $\{h\,o\,l\}$, figure 328; $-\frac{m\mathrm{P}\infty}{2}$ $\{o\,h\,l\}$, figure 326.

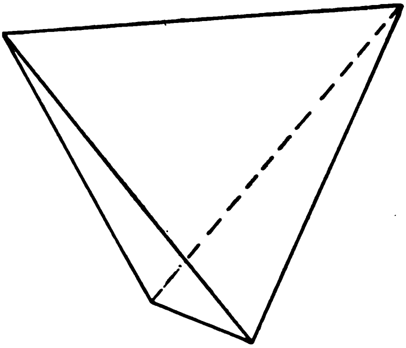

Fig. 329.

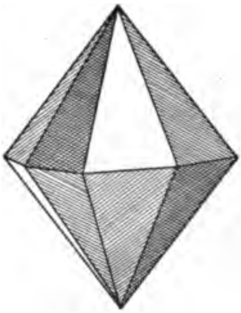

Fig. 330.

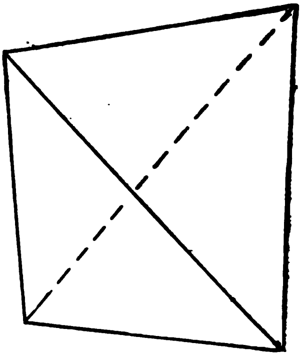

Fig. 331.

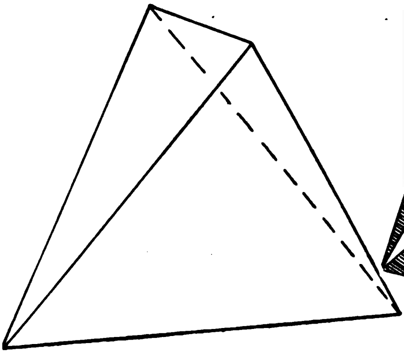

Fig. 332.

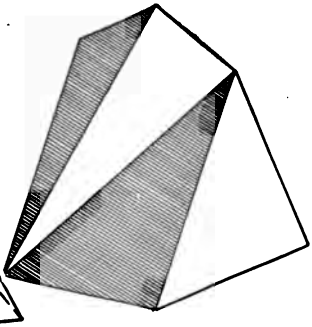

Fig. 333.

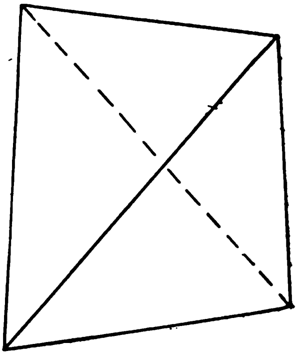

Fig. 334.

Bisphenoids of the third order. In a like manner, as is obvious, the ditetragonal bipyramid, figure 330, yields four *bisphenoids of the third order*. Figure 333 shows that these forms may also result when hemihedrism is applied to the tetragonal scalenohedron.

The symbols are:

Positive right, $+r\left[\dfrac{a : na : mc}{4}\right]$, $+r\dfrac{mPn}{4}$, $\{khl\}$, figure 321.

Positive left, $+l\left[\dfrac{a : na : mc}{4}\right]$, $+l\dfrac{mPn}{4}$, $\{hkl\}$, figure 329.

Negative right, $-r\left[\dfrac{a : na : mc}{4}\right]$, $-r\dfrac{mPn}{4}$, $\{k\bar{h}l\}$, figure 334.

Negative left, $-l\left[\dfrac{a : na : mc}{4}\right]$, $-l\dfrac{mPn}{4}$, $\{h\bar{k}l\}$, figure 332.

The same relationship exists between the bisphenoids of the three orders which was noted with the corresponding prisms, page 105.

In bisphenoids of the first order the secondary axes bisect the zigzag edges. They join the centers of opposite faces in forms of the second order, while in those of the third order they occupy an intermediate position.

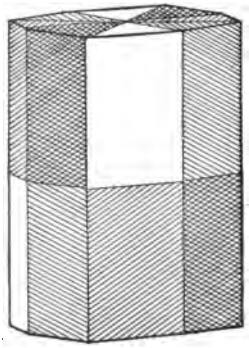

Fig. 335.

Prisms of the third order. Figure 335 shows that the ditetragonal prism now yields two prisms of the third order, compare figures 314 and 316.

The symbols are:

$\pm\left[\dfrac{a : na : \infty c}{2}\right]$; $+\dfrac{\infty Pn}{2}$ $\{hk0\}$; $-\dfrac{\infty Pn}{2}$ $\{h\bar{k}0\}$.

The other forms, prisms of the first and second orders and the basal pinacoid, remain unchanged.

The principal features may be tabulated as follows:

FORMS	SYMBOLS			Faces	Trihedral Solid Angles
	Weiss	Naumann	Miller		
Bisphenoids First order	$\pm\left[\frac{a:a:mc}{2}\right]$	$+\frac{mP}{2}$ $-\frac{mP}{2}$	$\{h\,h\,l\}$ $\{h\,\bar{h}\,l\}$	4	4
Bisphenoids Second order	$\pm\left[\frac{a:\infty a:mc}{2}\right]$	$+\frac{mP\propto}{2}$ $-\frac{mP\propto}{2}$	$\{h\,o\,l\}$ $\{o\,h\,l\}$	4	4
Bisphenoids Third order	$\pm\frac{r}{l}\left[\frac{a:na:mc}{4}\right]$	$+r\frac{mPn}{4}$ $-r\frac{mPn}{4}$ $+l\frac{mPn}{4}$ $-l\frac{mPn}{4}$	$\{k\,h\,i\}$ $\{k\,\bar{h}\,l\}$ $\{h\,k\,l\}$ $\{h\,\bar{k}\,l\}$	4	4
Prism First order	$a:a:\infty c$	Apparently holohedral			
Prism Second order	$a:\infty a:\infty c$				
Prisms Third order	$\pm\left[\frac{a:na:\infty c}{2}\right]$	$+\frac{mP\infty}{2}$ $-\frac{mP\infty}{2}$	$\{h\,k\,o\}$ $\{h\,\bar{k}\,o\}$	4	—
Pasal Pinacoid	$\infty a:\infty a:c$	Apparently holohedral			

No representatives of this class of symmetry have yet been observed.

ORTHORHOMBIC SYSTEM.1)

Crystallographic Axes. This system includes all forms which can be referred to three unequal and perpendicular axes, figure 336. One axis is held vertically, which is, as heretofore, the $\acute{c}$ axis. Another is directed toward the observer and is the $\breve{a}$ axis, sometimes also called the *brachydiagonal.* The third axis extends from right to left and is the $\bar{b}$ axis or *macrodiagonal.* There is no principal axis in this system, hence any axis may be chosen as the vertical or c axis. On this account one and the same crystal may be held in different positions by various observers, which has in some instances led to considerable confusion, for, as is obvious, the nomenclature of the various forms cannot then remain constant. In this system the axial ratio consists of two unknown values, viz: $\breve{a} : \bar{b} : \acute{c} = .8130 : 1 : 1.9037$, compare page 7.

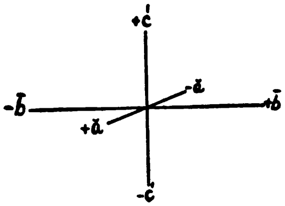

Fig. 336.

Classes of Symmetry. The orthorhombic system comprises three classes of symmetry, as follows:

1.	Orthorhombic bipyramidal class	(*Holohedrism.*)
2.	Orthorhombic pyramidal class	[*Holohedrism and hemimorphism.*]
3.	Orthorhombic bisphenoidal class	(*Hemihedrism.*)

Numerous representatives of all these classes have been observed among minerals and artificial salts. The first class is, however, the most important.

1. ORTHORHOMBIC BIPYRAMIDAL CLASS.2)

(*Holohedrism.*)

Symmetry. a) **Planes.** There are three planes of symmetry. These are perpendicular to each other and their intersection gives rise

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1) Sometimes termed the *trimetric*, *rhombic*, or *prismatic* system.

2) The *normal group* of Dana.

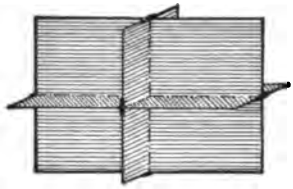


Fig. 337.

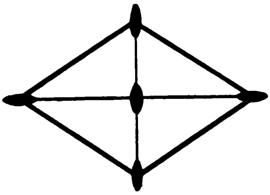


Fig. 338.

to the crystallographic axes, figure 337. Inasmuch as these planes are all dissimilar, they may be written:

$$1 + 1 + 1 = 3 \text{ planes.}$$

b) **Axes.** Three axes of binary symmetry are to be observed, figure 337. They are parallel to the crystallographic axes and indicated thus:

1● + 1● + 1● = 3 axes.

c) **Center.** This element of symmetry is also present and demands parallelism of faces. Figure 338 shows the above elements of symmetry.

Orthorhombic bipyramids. The form whose faces possess the parametral ratio, $\breve{a} : \bar{b} : \acute{c}$, is known as the *unit* or *fundamental orthorhombic bipyramid.* It consists of eight similar scalene triangles, figure 339. The symbols according to Naumann and Miller are as follows:

P, {111}.

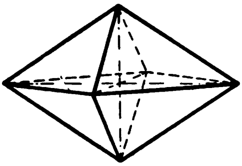


Fig. 339.

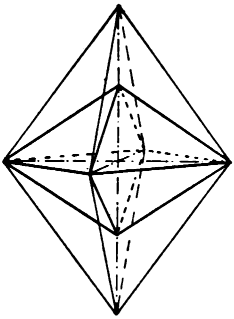


Fig. 340.

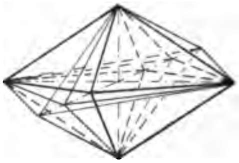


Fig. 341.

The outer form, in figure 340, possesses the ratio $\breve{a} : \bar{b} : m\acute{c}$ ($m > 0 < \infty$). In this case $m = 2$. This is a *modified orthorhombic bipyramid.* Its symbols are:

mP, $\{hhl\}$.

In figure 341, the heavy, inner form is the unit bipyramid. The lighter bipyramids intercept the $\bar{b}$ and $\acute{c}$ axes at unit distances but the

$\breve{a}$ axis at distances greater than unity. Their ratios may, however, be indicated in general as,

$$na : b : mc, \ (n > 1;\ m > 0 < \infty).$$

These are the *brachybipyramids*, because the intercepts along the *brachydiagonal* are greater than unity. The Naumann and Miller symbols are: $m\mathrm{P}\breve{n}$, $\{hkl\}$.

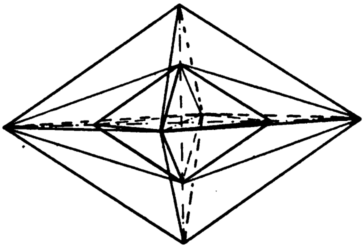


Fig. 342.

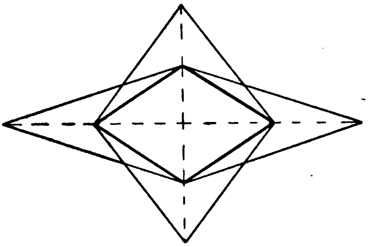


Fig. 343.

Figure 342 shows two bipyramids (outer) which cut the $\breve{a}$ axis at unity but intercept the $\bar{b}$ axis at the general distance nb, $(n > 1)$. The ratios would, therefore, be expressed by $a : nb : mc$. Since the intercepts along the *macrodiagonal* are greater than unity, these are called *macrobipyramids*. The symbols according to Naumann and Miller are $m\mathrm{P}\bar{n}$, $\{khl\}$.

Figure 343 shows the relationship existing between the *unit*, *macro-*, and *brachy bipyramids*, while figure 344 shows it for the *unit*, *modified*, and *macrobipyramids*.

Prisms. Similarly there are three types of prisms, namely, the *unit*, *macro-*, and *brachyprisms*. Each consists of four faces, cutting the $\breve{a}$ and $\bar{b}$ axes, but extending parallel to the $\acute{c}$ axis.

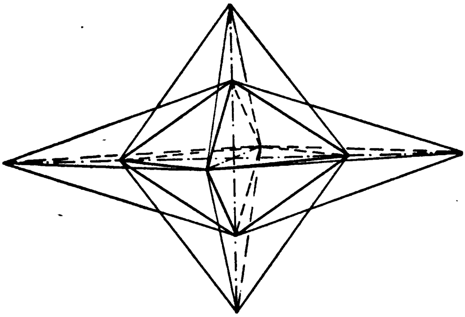


Fig. 344.

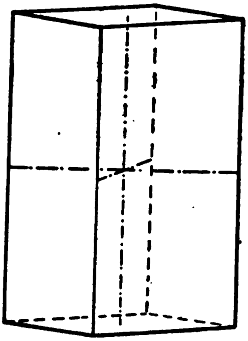


Fig. 345.

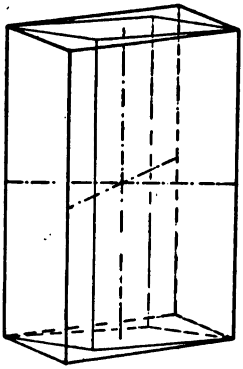


Fig. 346.

Figure 345 represents a *unit-prism* with the following symbols:

$\breve{a} : \bar{b} : \infty\dot{c}$, ∞P, $\{110\}$.

The *brachyprism* is shown in figure 346. Its symbols are:

$n\breve{a} : \bar{b} : \infty\dot{c}$, $\infty P\breve{n}$, $\{hk0\}$.

In figure 347, there is a unit prism surrounded by a *macroprism*, whose symbols may be written:

$\breve{a} : n\bar{b} : \infty\dot{c}$, $\infty P\bar{n}$, $\{kh0\}$.

For the relationship existing between these three prisms compare figure 343.

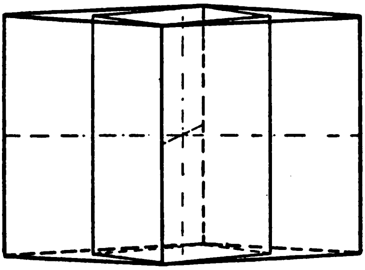


Fig. 347.

Domes. These are *horizontal prisms* and, hence, cut the $\dot{c}$ and one of the horizontal axes. Domes, which are parallel to the $\breve{a}$ axis or brachydiagonal, are called *brachydomes*. Their general symbols are:

$\infty\breve{a} : \bar{b} : m\dot{c}$, $mP\breve{\infty}$, $\{0hl\}$, figure 348.

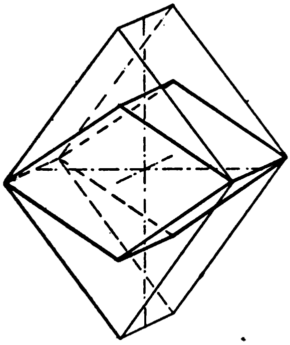


Fig. 348.

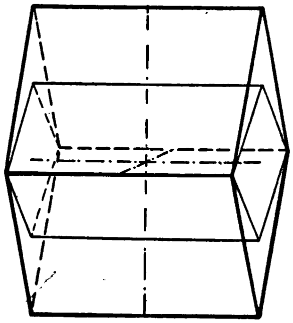


Fig. 349.

Those, which extend parallel to the macrodiagonal, are termed *macrodomes*, figure 349. Their symbols are:

$\breve{a} : \infty\bar{b} : m\dot{c}$, $mP\bar{\infty}$, $\{h0l\}$.

As is obvious, prisms and domes

are open forms and, hence, can only occur in combination with other forms.

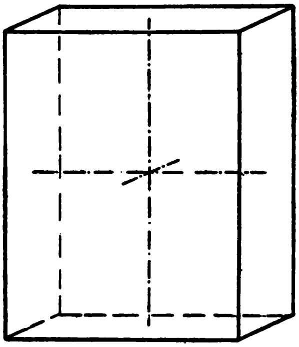


Fig. 350.

Pinacoids. These cut one axis and extend parallel to the other two. There are three types, as follows:

Basal pinacoids, $\infty\breve{a} : \infty\bar{b} : \acute{c}$, OP, {001}.

Brachypinacoids, $\infty\breve{a} : \bar{b} : \infty\acute{c}$, $\infty P\breve{\infty}$, {010}.

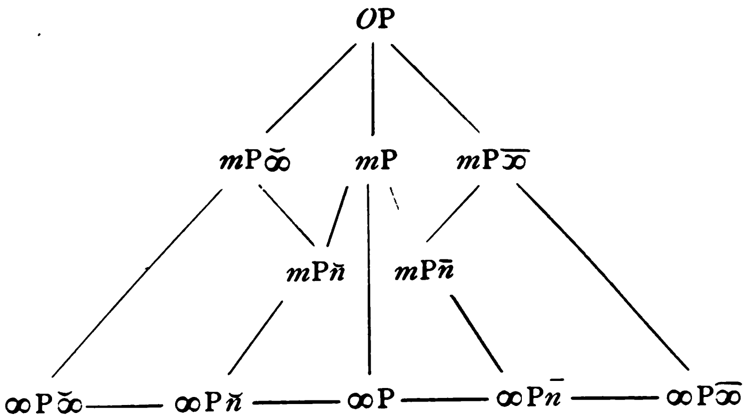
Macropinacoids, $\breve{a} : \infty\bar{b} : \infty\acute{c}$, $\infty P\bar{\infty}$, {100}.

These forms consist of two faces. Figure 350 shows a combination of three types of the pinacoids.

The characteristics of the forms of this class are given in the following table:

FORMS		SYMBOLS			Faces	Tetrahedral Solid Angles
		Weiss	Naumann	Miller		
Orthorhombic Bipyramids	Unit	$\breve{a} : \bar{b} : \acute{c}$	P	$\{111\}$	8	2+2+2
	Modified	$\breve{a} : \bar{b} : m\acute{c}$	mP	$\{hhl\}$		
	Brachy	$n\breve{a} : \bar{b} : m\acute{c}$	$mP\breve{n}$	$\{hkl\}$		
	Macro	$\breve{a} : n\bar{b} : m\acute{c}$	$mP\bar{n}$	$\{khl\}$		
Orthorhombic Prisms	Unit	$\breve{a} : \bar{b} : \infty\acute{c}$	∞P	$\{110\}$	4	
	Brachy	$n\breve{a} : \bar{b} : \infty\acute{c}$	$\infty P\breve{n}$	$\{hko\}$		
	Macro	$\breve{a} : n\bar{b} : \infty\acute{c}$	$\infty P\bar{n}$	$\{kho\}$		
Domes	Brachy	$\infty\breve{a} : \bar{b} : m\acute{c}$	$mP\breve{\infty}$	$\{ohl\}$	4	
	Macro	$\breve{a} : \infty\bar{b} : m\acute{c}$	$mP\bar{\infty}$	$\{hol\}$		
Pinacoids	Basal	$\infty\breve{a} : \infty\bar{b} : \acute{c}$	OP	$\{001\}$	2	
	Brachy	$\infty\breve{a} : \bar{b} : \infty\acute{c}$	$\infty P\breve{\infty}$	$\{010\}$		
	Macro	$\breve{a} : \infty\bar{b} : \infty\acute{c}$	$\infty P\bar{\infty}$	$\{100\}$		

Relationship of forms. This is to be seen from the following diagram:



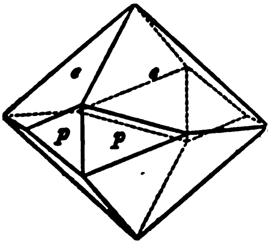


Fig. 351.

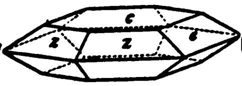


Fig. 352.

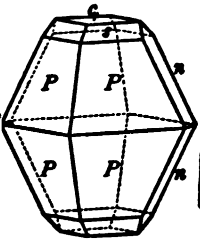


Fig. 353.

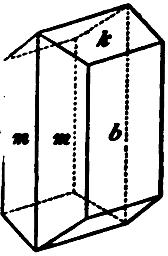


Fig. 354.

Combinations. Figure 351. $p = \infty P$, {110}; $e = P\breve{2}$, {122}. Brookite, TiO_2.

Figure 352. $c = OP$, {001}; $z = \frac{1}{3}P$, {113}; $e = \frac{2}{3}P\breve{\infty}$, {023}. Chalcocite, Cu_2S.

Figure 353. $P = P$, {111}; $s = \frac{1}{3}P$, {113}; $c = OP$, {001}. Sulphur, S.

Figure 354. $m = \infty P$, {110}; $b = \infty P\breve{\infty}$, {010}; $k = P\breve{\infty}$, {011}. Aragonite, $CaCO_3$.

Figure 355. $a = \infty P\bar{\infty}$, {100}; $s = \infty P\breve{2}$, {120}; $b = \infty P\breve{\infty}$, {010}; $o = P$, {111}; $n = 2P\bar{2}$, {211}; $i = P\breve{\infty}$, {011}. Chrysoberyl, $Be(AlO_2)_2$.

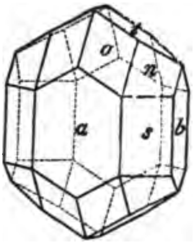


Fig. 355.

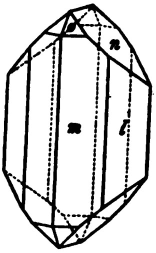


Fig. 356.

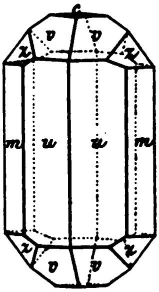


Fig. 357.

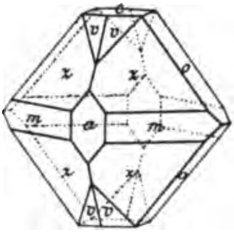


Fig. 358.

Figure 356. $m = \infty P$, $\{110\}$; $l = \infty P2$, $\{120\}$; $n = 2P\breve{\infty}$, $\{021\}$; $o = P$, $\{111\}$. Topaz, $Al_2(F.OH)_2SiO_4$.

Figures 357 to 359. $m = \infty P$, $\{110\}$; $u = \infty P\bar{\tfrac{3}{2}}$, $\{320;\}$ $a = \infty P\bar{\infty}$, $\{100\}$; $z = P$, $\{111\}$; $v = \tfrac{3}{4}P\bar{\tfrac{3}{2}}$, $\{324\}$; $c = OP$, $\{001\}$; $o = \infty P\breve{\infty}$, $\{011\}$; $d = \tfrac{1}{2}P\bar{\infty}$, $\{102\}$; $x = P\bar{4}$, $\{414\}$; $y = P\breve{2}$ $\{122\}$. Celestite, $SrSO_4$.

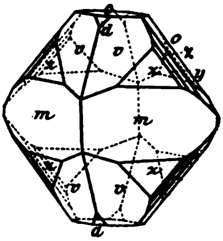


Fig. 359.

2. *ORTHORHOMBIC PYRAMIDAL CLASS.*[1]

(*Holohedrism with Hemimorphism.*)

Symmetry. As figure 360 shows, the elements of symmetry of this class are the vertical planes of symmetry and one axis of binary symmetry parallel to the $\dot{c}$ axis. This axis of symmetry is obviously polar.

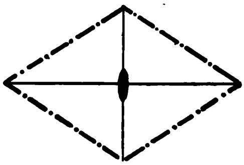


Fig. 360.

All forms with the exception of the prisms, the macro- and brachypinacoids are now divided into *upper* and *lower* forms, as shown in the following tabulation:

---

[1] The *hemimorphic group* of Dana.

FORMS		SYMBOLS			Faces	Tetrahedral Solid Angles
		Weiss	Naumann	Miller		
Orthorhombic Pyramids	Unit	$\left[\frac{\breve{a}:\bar{b}:\dot{c}}{2}\right]\frac{u}{l}$	$\frac{P}{2}u$ $\frac{P}{2}l$	$\{111\}$ $\{11\bar{1}\}$	4	1
	Modified	$\left[\frac{\breve{a}:\bar{b}:m\dot{c}}{2}\right]\frac{u}{l}$	$\frac{mP}{2}u$ $\frac{mP}{2}l$	$\{hhl\}$ $\{hh\bar{l}\}$		
	Brachy	$\left[\frac{n\breve{a}:\bar{b}:m\dot{c}}{2}\right]\frac{u}{l}$	$\frac{mP\breve{n}}{2}u$ $\frac{mP\breve{n}}{2}l$	$\{hkl\}$ $\{hk\bar{l}\}$		
	Macro	$\left[\frac{\breve{a}:n\bar{b}:m\dot{c}}{2}\right]\frac{u}{l}$	$\frac{mP\bar{n}}{2}u$ $\frac{mP\bar{n}}{2}l$	$\{khl\}$ $\{kh\bar{l}\}$		
Orthorhombic Prisms	Unit	$\breve{a}:\bar{b}:\infty\dot{c}$	∞P	$\{110\}$	Apparently holohedral	
	Brachy	$n\breve{a}:\bar{b}:\infty\dot{c}$	$\infty P\breve{n}$	$\{hk0\}$		
	Macro	$\breve{a}:n\bar{b}:\infty\dot{c}$	$\infty P\bar{n}$	$\{kh0\}$		
Domes	Brachy	$\left[\frac{\infty\breve{a}:\bar{b}:m\dot{c}}{2}\right]\frac{u}{l}$	$\frac{mP\breve{\infty}}{2}u$ $\frac{mP\breve{\infty}}{2}l$	$\{0hl\}$ $\{0h\bar{l}\}$	2	—
	Macro	$\left[\frac{\breve{a}:\infty\bar{b}:m\dot{c}}{2}\right]\frac{u}{l}$	$\frac{mP\bar{\infty}}{2}u$ $\frac{mP\bar{\infty}}{2}l$	$\{h0l\}$ $\{h0\bar{l}\}$		
Pinacoids	Basal	$\left[\frac{\infty\breve{a}:\infty\bar{b}:\dot{c}}{2}\right]\frac{u}{l}$	$\frac{OP}{2}u$ $\frac{OP}{2}l$	$\{001\}$ $\{00\bar{1}\}$	1	—
	Brachy	$\infty\breve{a}:\bar{b}:\infty\dot{c}$	$\infty P\breve{\infty}$	$\{010\}$	Apparently holohedral	
	Macro	$\breve{a}:\infty\bar{b}:\infty\dot{c}$	$\infty P\bar{\infty}$	$\{100\}$		

Combinations. Figure 361. $a = \infty P\bar{\infty}$, $\{100\}$; $g = \infty P$, $\{110\}$; $b = \infty P\breve{\infty}$, $\{010\}$; $o = \frac{P\bar{\infty}}{2}u$, $\{101\}$; $p = \frac{3P\bar{\infty}}{2}u$, $\{301\}$; $m = \frac{3P\breve{\infty}}{2}u$, $\{031\}$; $r = \frac{P\breve{\infty}}{2}u$, $\{011\}$; $c = OPu$, $\{001\}$; $s = 2\breve{P}2l$, $\{12\bar{1}\}$. Calamine, also called hemimorphite, $Zn_2(OH)_2SiO_3$.

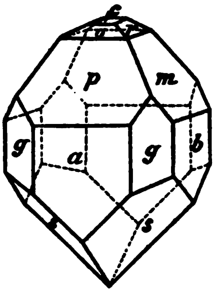


Fig. 361.

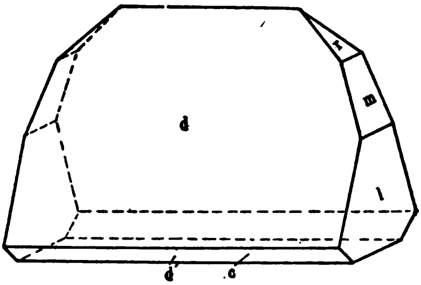


Fig. 362.

Figure 362. $d = \frac{P\infty}{2}u$, $\{101\}$; $r = \frac{P\breve{\infty}}{2}u$, $\{011\}$; $m = \frac{4P\breve{\infty}}{2}u$, $\{041\}$; $l = \infty P\breve{\infty}$, $\{010\}$; $d' = \frac{\frac{1}{3}P\bar{\infty}}{2}l$, $\{10\bar{3}\}$; $c = \frac{OP}{2}l$, $\{00\bar{1}\}$. Struvite, $NH_4MgPO_4+6H_2O$.

3. ORTHORHOMBIC BISPHENOIDAL CLASS.[1]

(*Sphenoidal Hemihedrism.*)

Symmetry. As can be seen from figure 363, this class possesses three axes of binary symmetry. The other elements have disappeared. The method of extension and suppression of faces is comparable to the tetrahedral and sphenoidal hemihedrisms of the cubic and tetragonal systems, respectively, in that faces lying wholly within alternate octants are extended.

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[1] Dana terms this class the *sphenoidal group*.

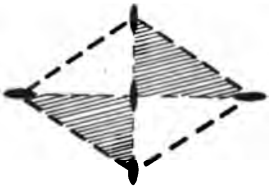

Fig. 363.

Orthorhombic bisphenoids. Figure 365 shows the application of the sphenoidal hemihedrism to an orthorhombic bipyramid, which yields two new correlated, enantiomorphous forms, called the *right*, figure 366, and *left*, figure 364, *orthorhombic bisphenoids*. These forms are bounded by four scalene triangles. The crystallographic axes bisect the edges.

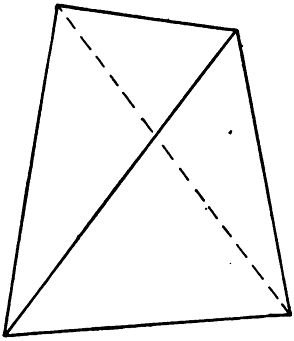

Fig. 364.

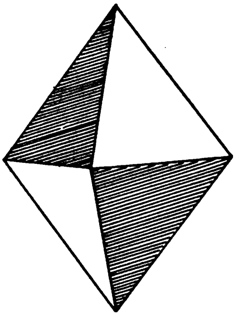

Fig. 365.

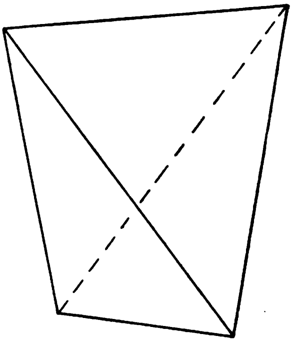

Fig. 366.

Since there are several types of orthorhombic bipyramids, it follows that each will yield two enantiomorphous bisphenoids. They are termed the *orthorhombic bisphenoids*. These are given and their symbols indicated in the tabulation on page 125.

All other forms must occur with the full number of faces, that is, they are apparently holohedral.

FORMS		SYMBOLS			Faces	Trihedral Solid Angles
		Weiss	Naumann	Miller		
Orthorhombic Bisphenoids	Unit	$r, l\left[\frac{\breve{a}:\bar{b}:\dot{c}}{2}\right]$	$r\frac{P}{2}$ $l\frac{P}{2}$	$\{111\}$ $\{1\bar{1}1\}$	4	4
	Modified	$r, l\left[\frac{\breve{a}:\bar{b}:m\dot{c}}{2}\right]$	$r\frac{mP}{2}$ $l\frac{mP}{2}$	$\{hhl\}$ $\{h\bar{h}l\}$		
	Brachy	$r, l\left[\frac{n\breve{a}:\bar{b}:m\dot{c}}{2}\right]$	$r\frac{mP\breve{n}}{2}$ $l\frac{mP\breve{n}}{2}$	$\{hkl\}$ $\{h\bar{k}l\}$		
	Macro	$r, l\left[\frac{\breve{a}:n\bar{b}:m\dot{c}}{2}\right]$	$r\frac{mP\bar{n}}{2}$ $l\frac{mP\bar{n}}{2}$	$\{khl\}$ $\{k\bar{h}l\}$		
Orthorhombic Prisms	Unit	$\breve{a}:\bar{b}:\infty\dot{c}$	∞P	$\{110\}$	Apparently holohedral	
	Brachy	$n\breve{a}:\bar{b}:\infty\dot{c}$	$\infty P\breve{n}$	$\{hk0\}$		
	Macro	$\breve{a}:n\bar{b}:\infty\dot{c}$	$\infty P\bar{n}$	$\{kh0\}$		
Domes	Brachy	$\infty\breve{a}:\bar{b}:m\dot{c}$	$mP\breve{\infty}$	$\{0hl\}$		
	Macro	$\breve{a}:\infty\bar{b}:m\dot{c}$	$mP\bar{\infty}$	$\{h0l\}$		
Pinacoids	Basal	$\infty\breve{a}:\infty\bar{b}:\dot{c}$	$0P$	$\{001\}$		
	Brachy	$\infty\breve{a}:\bar{b}:\infty\dot{c}$	$\infty P\breve{\infty}$	$\{010\}$		
	Macro	$\breve{a}:\infty\bar{b}:\infty\dot{c}$	$\infty P\bar{\infty}$	$\{100\}$		

Combination. Figure 367. $m = \infty P$, $\{110\}$; $l = l\frac{P}{2}$, $\{1\bar{1}1\}$; $z = r\frac{P}{2}$, $\{111\}$. Epsomite, $MgSO_4 + 7H_2O$.

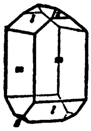

Fig. 367.

MONOCLINIC SYSTEM.(1)

Crystallographic Axes. To this system belong those forms which can be referred to three unequal axes, two of which ($\acute{a}$ and $\acute{c}$) intersect at an oblique angle, while the third axis ($\bar{b}$) is perpendicular to these two. The oblique angle between the $\acute{a}$ and $\acute{c}$ axes is termed β. Figure 368 shows an axial cross of this system.

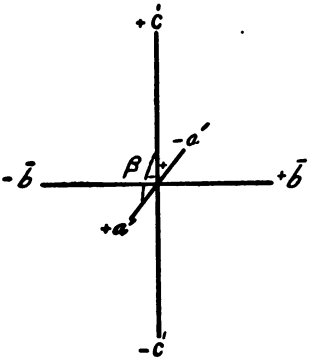

Fig. 368.

It is customary to place the $\bar{b}$ axis so as to extend from right to left. The $\acute{c}$ axis is held vertically. The $\acute{a}$ axis is then directed toward the observer. Since the $\acute{a}$ axis is inclined, it is called the *clino-axis*. The $\bar{b}$ axis is often spoken of as the *ortho-axis*. The obtuse angle between the $\acute{a}$ and $\acute{c}$ axis is the *negative* angle β, whereas the acute angle is *positive*. Obviously, they are supplementary angles. The elements of crystallization consist of the axial ratio and the angle β, which may be either the obtuse or the acute angle. Compare page 8.

Classes of Symmetry. The monoclinic system includes three classes of symmetry, as follows:

1. Prismatic class (*Holohedrism.*)
2. Domatic class (*Hemihedrism.*)
3. Sphenoidal class (*Hemimorphism.*)

The first class is the most important.

1. MONOCLINIC PRISMATIC CLASS.(2)

(*Holohedrism.*)

Symmetry. This class possesses one plane of symmetry, which includes the $\acute{a}$ and $\acute{c}$ axes and, hence, is directed toward the observer.

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(1) Also termed the clinorhombic, hemiprismatic, monoclinohedral, monosymmetric or oblique system.

(2) *Normal group* of Dana.

Perpendicular to this plane, that is, parallel to the $\bar{b}$ axis, is an axis of binary symmetry. A center of symmetry is also present. Figure 369 shows these elements in a crystal of augite. These elements are represented diagrammatically in figure 370, which is a projection of a monoclinic form upon the plane of the a' and $\bar{b}$ axes.

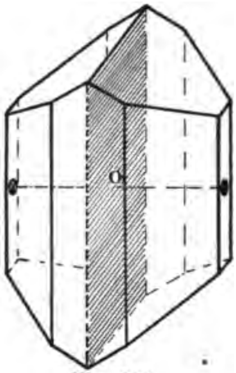

Fig. 369.

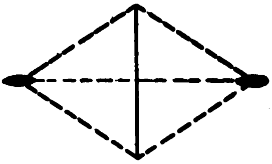

Fig. 370.

Hemi-pyramids. On account of the presence in this class of only one plane of symmetry and an axis of binary symmetry, a form with unit intercepts, that is, with the parametral ratio $a' : \bar{b} : \acute{c}$, can possess but four faces. Figure 371 shows four such faces, which enclose the positive angle β and are said to constitute the *positive unit hemi-pyramid.*1) Figure 372 shows four faces with the same ratio enclosing the negative angle β, and comprising the *negative unit hemi-pyramid.* It is obvious that the faces of these hemi-pyramids are dissimilar, those over the negative angle being the larger. The symbols for these hemi-pyramids are according to Naumann and Miller, + P $\{11\bar{1}\}$ and — P $\{111\}$, respectively. Two unit hemi-pyramids occurring simultaneously constitute the *monoclinic unit bipyramid*, figure 373.

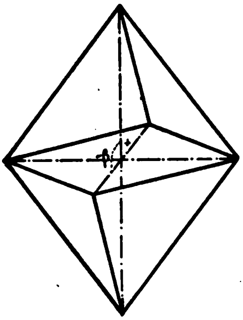

Fig. 371.

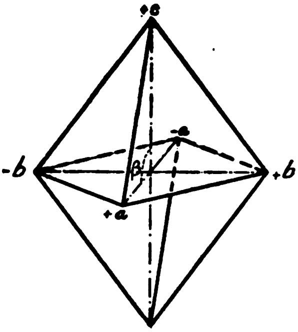

Fig. 372.

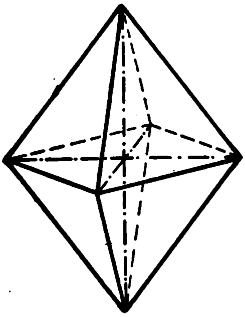

Fig. 373.

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1) The term *hemi-bipyramid* ought more properly be employed because of the fact that we are dealing with four faces of a bipyramid and not of a pyramid as the term has hitherto been used.

Since this system differs essentially from the orthorhombic in the obliquity of the a axis, it follows that *modified*, *clino*, and *ortho* hemi-pyramids are also possible. They possess the following general symbols:

Modified hemi-pyramids,
$\pm(a' : \bar{b} : m\grave{c})$, $m > 0 < \infty$; $+m\mathrm{P}\{h h \bar{l}\}$; $-m\mathrm{P}\}h h l\}$.
Clino hemi-pyramids,
$\pm(na' : \bar{b} : m\grave{c})$, $n > 1$; $+m\mathrm{P}n'$ $\{h k \bar{l}\}$; $-m\mathrm{P}n'$ $\{h k l\}$, $h < k$.
Ortho hemi-pyramids,
$+(a' : n\bar{b} : m\grave{c})$, $n > 1$; $+m\mathrm{P}\bar{n}$ $\{h k \bar{l}\}$; $-m\mathrm{P}\bar{n}$ $\{h k l\}$; $h > k$.

Prisms. As was the case in the orthorhombic system, page 117, there are also three types of prisms possible in this system, namely, *unit*, *clino-*, and *ortho-prisms*. These forms cut the a' and $\bar{b}$ axes and extend parallel to the vertical axis.

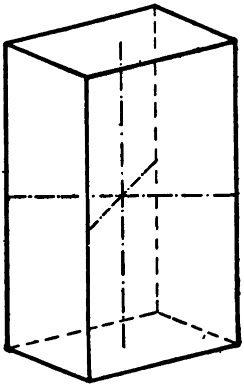

Fig. 374.

The general symbols are:

Unit prism,
$a' : \bar{b} : \infty\grave{c}$, $\infty\mathrm{P}$, $\{110\}$, figure 374.
Clinoprism,
$na' : \bar{b} : \infty\grave{c}$, $\infty\mathrm{P}n'$, $\{h k o\}$; $n > 1$, $h < k$.
Orthoprism,
$a' : n\bar{b} : \infty\grave{c}$, $\infty\mathrm{P}\bar{n}$, $\{h k o\}$; $n > 1$, $h > k$.

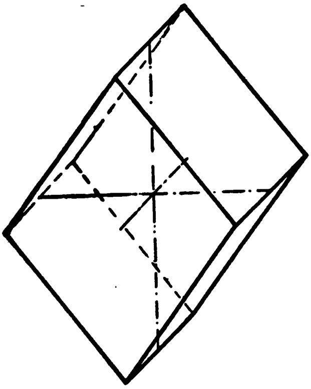

Fig. 375.

Domes. In this system two types of domes are also possible, namely, those which extend parallel to the a' and $\bar{b}$ axes, respectively. Those, which are parallel to a', are termed *clinodomes* and consist of four faces, figure 375. The general symbols are:

$\infty a' : \bar{b} : m\grave{c}$, $m\mathrm{P}\infty'$, $\{o h l\}$.

Since the a' axis is inclined to the $\grave{c}$, it follows that the domes which are parallel to the $\bar{b}$ axis consist of but two faces. Figure 376 shows such faces enclosing the positive angle and are termed the *positive hemi-orthodome*, whereas in 377 the *negative hemi-orthodome* is

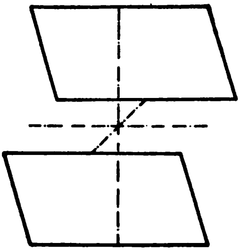

Fig. 376.

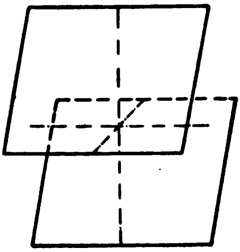

Fig. 377.

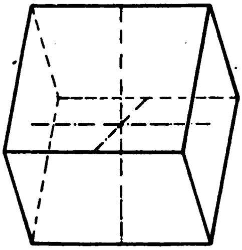

Fig. 378.

represented. It is evident that the faces of the positive form are always the smaller. Figure 378 shows these hemidomes in combination. Their general symbols are:

Positive hemi-orthodome,

$$a' : \infty\bar{b} : m\acute{c},\ +m\text{P}\bar{\infty},\ \{h0\bar{l}\}.$$

Negative hemi-orthodome,

$$a' : \infty\bar{b} : m\acute{c},\ -m\text{P}\bar{\infty},\ \{h0l\}.$$

Pinacoids. There are three types of pinacoids possible in the monoclinic system, namely,

Basal pinacoids, $\infty a' : \infty\bar{b} : \acute{c}$, OP, {001}.

Clinopinacoids, $\infty a' : \bar{b} : \infty\acute{c}$, $\infty\text{P}\infty'$, {010}.

Orthopinacoids, $a' : \infty\bar{b} : \infty\acute{c}$, $\infty\text{P}\bar{\infty}$, {100}.

These are forms consisting of but two faces. Figure 379 shows a combination of these pinacoids.

All forms of the monoclinic system are open forms and, hence, every crystal of this system is a combination.

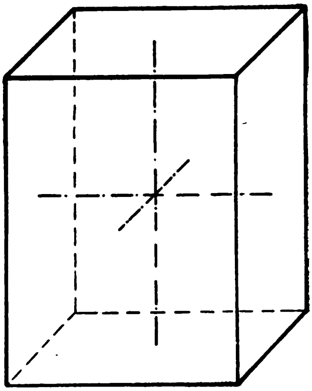

Fig. 379.

A summary of the forms of this class is given as follows:

FORMS		SYMBOLS			Faces
		Weiss	Naumann	Miller	
Hemi-pyramids	Unit	$\pm(a':\bar{b}:\dot{c})$	$+P$ $-P$	$\{11\bar{1}\}$ $\{111\}$	4
	Modified	$\pm(a':\bar{b}:m\dot{c})$	$+mP$ $-mP$	$\{hh\bar{l}\}$ $\{hhl\}$	
	Clino-	$\pm(na':\bar{b}:m\dot{c})$	$+mPn'$ $-mPn'$	$\{hk\bar{l}\}$ $\{hkl\}$	
	Ortho-	$\pm(a':n\bar{b}:m\dot{c})$	$+mP\bar{n}$ $-mP\bar{n}$	$\{kh\bar{l}\}$ $\{khl\}$	
Prisms	Unit	$a':\bar{b}:\infty\dot{c}$	∞P	$\{110\}$	4
	Clino-	$na':\bar{b}:\infty\dot{c}$	$\infty Pn'$	$\{hko\}$	
	Ortho-	$a':n\bar{b}:\infty\dot{c}$	$\infty P\bar{n}$	$\{kho\}$	
Clinodome		$\infty a':\bar{b}:m\dot{c}$	$mP\infty'$	$\{ohl\}$	4
Hemi-orthodomes	Positive	$a':\infty\bar{b}:m\dot{c}$	$+mP\bar{\infty}$	$\{ho\bar{l}\}$	2
	Negative	$a':\infty\bar{b}:m\dot{c}$	$-mP\bar{\infty}$	$\{hol\}$	
Pinacoids	Basal	$\infty a':\infty\bar{b}:\dot{c}$	OP	$\{001\}$	2
	Clino-	$\infty a':\bar{b}:\infty\dot{c}$	$\infty P\infty'$	$\{010\}$	
	Ortho-	$a':\infty\bar{b}:\infty\dot{c}$	$\infty P\bar{\infty}$	$\{100\}$	

Relationship of forms. This is clearly shown by the following diagram:

$$\begin{tikzcd}
OP \arrow[d, dash] & OP \arrow[d, dash] & OP \arrow[d, dash] & OP \arrow[d, dash] & OP \arrow[d, dash] \\
\frac{1}{m}P\infty' \arrow[r, dash] \arrow[d, dash] & \pm\frac{1}{m}Pn' \arrow[r, dash] \arrow[d, dash] & \pm\frac{1}{m}P \arrow[r, dash] \arrow[d, dash] & \pm\frac{1}{m}P\bar{n} \arrow[r, dash] \arrow[d, dash] & \pm\frac{1}{m}P\bar{\infty} \arrow[d, dash] \\
P\infty' \arrow[r, dash] \arrow[d, dash] & \pm Pn' \arrow[r, dash] \arrow[d, dash] & \pm P \arrow[r, dash] \arrow[d, dash] & \pm P\bar{n} \arrow[r, dash] \arrow[d, dash] & \pm P\bar{\infty} \arrow[d, dash] \\
mP\infty' \arrow[r, dash] \arrow[d, dash] & \pm mPn' \arrow[r, dash] \arrow[d, dash] & \pm mP \arrow[r, dash] \arrow[d, dash] & \pm mP\bar{n} \arrow[r, dash] \arrow[d, dash] & \pm mP\bar{\infty} \arrow[d, dash] \\
\infty P\infty' \arrow[r, dash] & \pm\infty Pn \arrow[r, dash] & \infty P \arrow[r, dash] & \pm\infty P\bar{n} \arrow[r, dash] & \infty P\bar{\infty}
\end{tikzcd}$$

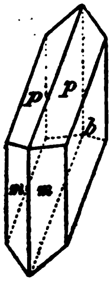

Fig. 380.

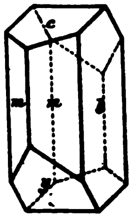

Fig. 381.

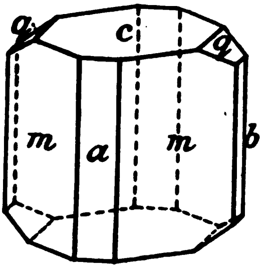

Fig. 382.

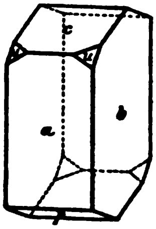

Fig. 383.

Combinations. Figure 380. $m = \infty P$, {110}; $b = \infty P\infty'$, {010}; $p = -P$, {111}. Gypsum, $CaSO_4.2H_2O$.

Figure 381. $c = OP$, {001}; $m = \infty P$, {110}; $b = \infty P\infty'$, {010}; $y = 2P\bar{\infty}$, {$20\bar{1}$}. Orthoclase, $KAlSi_3O_8$.

Figure 382. $m = \infty P$, {110}; $c = OP$, {001}; $a = \infty P\bar{\infty}$, {100}; $q = P\infty'$, {011}. Thorium sulphate, $Th(SO_4)_2.9H_2O$.

Figure 383. $c = OP$, {001}; $a = \infty P\bar{\infty}$, {100}; $b = \infty P\infty'$, {010}; $u = -P$, {111}. Diopside, $CaMg(SiO_3)_2$.

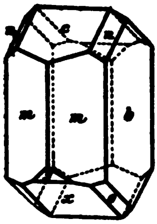

Fig. 384.

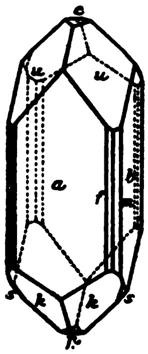

Fig. 385.

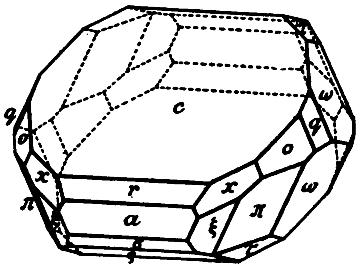

Fig. 386.

Figure 384. $m = \infty P$, {110}; $b = \infty P\infty'$, {010}; $c = OP$, {001}; $n = 2P\infty'$, {021}; $y = 2P\bar{\infty}$, {$20\bar{1}$}; $o = P$, {$11\bar{1}$}; $x = P\bar{\infty}$, {$10\bar{1}$}. Orthoclase.

Figure 385. $a = \infty P\bar{\infty}$, {100}; $b = \infty P\infty'$, {010}; $u = -P$, {111}; $c = OP$, {001}; $m = \infty P$, {110}; $f = \infty P\bar{3}$, {310}. Diopside.

Figure 386. $c = OP$, {001}; $a = \infty P\bar{\infty}$, {100}; $r = P\bar{\infty}$, {101}; $o = -P$, {111}; $\omega = +P$, {$11\bar{1}$}; $\varrho = +P\infty$, {$10\bar{1}$};

$\sigma = 2P\bar{\infty}$, $\{20\bar{1}\}$; $q = P\infty'$, $\{011\}$; $\pi = 2P\bar{2}$, $\{21\bar{1}\}$; $\xi = +3P\bar{3}$, $\{3\bar{1}\bar{1}\}$; $= P3$, $\{31\bar{3}\}$; $x = -3P\bar{3}$, $\{311\}$. Praseodymium sulphate, $Pr_2(SO_4)_3.8H_2O$.

2. MONOCLINIC DOMATIC CLASS.[1]

(*Hemihedrism.*)

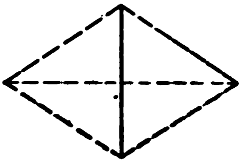

Fig. 387.

Symmetry. This class possesses but one element of symmetry, namely, a plane of symmetry as shown in figure 387.

Tetra-pyramids. Since the axis of binary symmetry is lost, it follows that the hemi-pyramids now yield two *tetra-pyramids.* These forms consist of but two faces which are situated symmetrically to the plane of symmetry. For example, in figure 371 the positive hemi-pyramid yields the *lower positive tetra-pyramid,* which consists of the two faces in front, and the *upper positive tetra-pyramid* made up of the other two faces. In like manner the negative hemi-pyramid yields the *upper* and *lower negative tetra-pyramids.* Their symbols are given in the tabulation on page 133.

Hemi-clinodomes. Obviously, the clinodome is now divided into an *upper* and a *lower hemi-clinodome.*

Tetra-domes. The hemi-orthodomes yield *upper* and *lower tetra-domes.*

Hemi-prisms. Each prism now yields two *hemi-prisms,* designated as *front* and *rear* forms.

Pinacoids. The clinopinacoids must on account of the symmetry occur with the full number of faces, namely, two. The basal and orthopinacoids each yield two forms. In the case of the basal pinacoid, they are designated as *upper* and *lower* forms, while the terms *front* and *rear* are employed for the orthopinacoids.

The forms and their general symbols may be tabulated as follows:

---

[1] *Clinohedral group* of Dana.

FORMS		SYMBOLS			Faces
		Weiss	Naumann	Miller	
Tetra-pyramids	Unit and Modified	$\pm\left[\frac{a':\bar{b}:m\grave{c}}{2}\right]u, l$	$+\frac{mP}{2}l$	$\{hh\bar{l}\}$	2
			$+\frac{mP}{2}u$	$\{\bar{h}hl\}$	
			$-\frac{mP}{2}u$	$\{hhl\}$	
			$-\frac{mP}{2}l$	$\{\bar{h}h\bar{l}\}$	
	Clino	$\pm\left[\frac{na':\bar{b}:m\grave{c}}{2}\right]u, l$	$+\frac{mPn'}{2}l$	$\{hk\bar{l}\}$	
			$+\frac{mPn'}{2}u$	$\{\bar{h}kl\}$	
			$-\frac{mPn'}{2}u$	$\{hkl\}$	
			$-\frac{mPn'}{2}l$	$\{\bar{h}k\bar{l}\}$	
	Ortho	$\pm\left[\frac{a':n\bar{b}:m\grave{c}}{2}\right]u, l$	$+\frac{mP\bar{n}}{2}l$	$\{kh\bar{l}\}$	
			$+\frac{mP\bar{n}}{2}u$	$\{\bar{k}hl\}$	
			$-\frac{mP\bar{n}}{2}u$	$\{khl\}$	
			$-\frac{mP\bar{n}}{2}l$	$\{\bar{k}h\bar{l}\}$	
Hemi-prisms	Unit	$\left[\frac{a':\bar{b}:\infty\grave{c}}{2}\right]f, r$	$\frac{\infty P}{2}f$	$\{110\}$	2
			$\frac{\infty P}{2}r$	$\{\bar{1}10\}$	
	Clino	$\left[\frac{na':\bar{b}:\infty\grave{c}}{2}\right]f, r$	$\frac{\infty Pn'}{2}f$	$\{hk0\}$	
			$\frac{\infty Pn'}{2}r$	$\{\bar{h}k0\}$	
	Ortho	$\left[\frac{a':n\bar{b}:\infty\grave{c}}{2}\right]f, r$	$\frac{\infty P\bar{n}}{2}f$	$\{kh0\}$	
			$\frac{\infty P\bar{n}}{2}r$	$\{\bar{k}h0\}$	

FORMS		SYMBOLS			Faces
		Weiss	Naumann	Miller	
Hemi-clinodomes		$\left[\frac{\infty a' : \bar{b} : m\grave{c}}{2}\right] u, l$	$\frac{mP\infty'}{2} u$ $\frac{mP\infty'}{2} l$	$\{0hl\}$ $\{0hl$	2
Tetra-orthodomes		$\pm\left[\frac{a' : \infty\bar{b} : m\grave{c}}{2}\right] u, l$	$+\frac{mP\overline{\infty}}{2} u$ $+\frac{mP\overline{\infty}}{2} l$ $-\frac{mP\overline{\infty}}{2} u$ $-\frac{mP\overline{\infty}}{2} l$	$\{\bar{h}0l\}$ $\{h0\bar{l}\}$ $\{h0l\}$ $\{\bar{h}0\bar{l}\}$	1
Clinopinacoid		$\infty a' : \bar{b} : \infty\grave{c}$	Apparently holohedral		
Pinacoids	Basal	$\left[\frac{\infty a' : \infty\bar{b} : \grave{c}}{2}\right] u, l$	$\frac{0P}{2} u$ $\frac{0P}{2} l$	$\{001\}$ $\{00\bar{1}\}$	1
	Ortho	$\left[\frac{a' : \infty\bar{b} : \infty\grave{c}}{2}\right] f, r$	$\frac{\infty P\overline{\infty}}{2} f$ $\frac{\infty P\overline{\infty}}{2} r$	$\{100\}$ $\{\bar{1}00\}$	

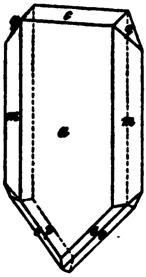

Fig. 388.

Combination. Fig. 388. $c = \frac{0P}{2} u$, $\{001\}$; $a = \frac{\infty P\overline{\infty}}{2} f$, $\{100\}$, and $\frac{\infty P\overline{\infty}}{2} r$, $\{\bar{1}00\}$; $m = \frac{\infty P}{2} f$, $\{110\}$, and $\frac{\infty P}{2} r$, $\{\bar{1}10\}$; $q = \frac{P\infty'}{2} u$, $\{011\}$; $v = +\frac{P3'}{2} l$, $\{13\bar{3}\}$; $o = +\frac{P}{2} l$, $\{11\bar{1}\}$. Tetrathionate of potassium, $K_2S_4O_6$.

3. MONOCLINIC SPHENOIDAL CLASS.[1]

(*Hemimorphism.*)

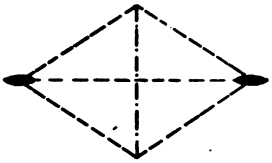

Fig. 389.

Symmetry. A polar axis of binary symmetry is the only element of symmetry present, figure 389. The forms of this class are hemimorphic along the *b* axis, which is now a singular axis.

Those forms which extend parallel to $\bar{b}$ axis, the orthodomes, basal and orthopinacoids, are unaltered. The others yield *right* and *left* forms, for example, the unit prism yields the *right* and *left hemi-prisms*.

The forms and their symbols are tabulated as follows:

FORMS		SYMBOLS			Faces
		Weiss	Naumann	Miller	
Tetra-pyramids (Sphenoids)	Unit and Modified	$\pm r, l\left[\frac{a' : \bar{b} : m\grave{c}}{2}\right]$	$+r\frac{mP}{2}$	$\{hh\bar{l}\}$	2
			$+l\frac{mP}{2}$	$\{h\bar{h}\bar{l}\}$	
			$-r\frac{mP}{2}$	$\{hhl\}$	
			$-l\frac{mP}{2}$	$\{h\bar{h}l\}$	
	Clino	$\pm r, l\left[\frac{na' : \bar{b} : m\grave{c}}{2}\right]$	$+r\frac{mPn'}{2}$	$\{hk\bar{l}\}$	
			$+l\frac{mPn'}{2}$	$\{h\bar{k}\bar{l}\}$	
			$-r\frac{mPn'}{2}$	$\{hkl\}$	
			$-l\frac{mPn'}{2}$	$\{h\bar{k}l\}$	
	Ortho	$\pm r, l\left[\frac{na' : n\bar{b} : m\grave{c}}{2}\right]$	$+r\frac{mP\bar{n}}{2}$	$\{kh\bar{l}\}$	
			$+l\frac{mP\bar{n}}{2}$	$\{k\bar{h}\bar{l}\}$	
			$-r\frac{mP\bar{n}}{2}$	$\{khl\}$	
			$-l\frac{mP\bar{n}}{2}$	$\{k\bar{h}l\}$	

---

[1] Dana terms this class the *hemimorphic group*.

FORMS		SYMBOLS			Faces
		Weiss	Naumann	Miller	
Hemi-prisms	Unit	$r, l\left[\frac{a' : \bar{b} : \infty \dot{c}}{2}\right]$	$r \frac{\infty P}{2}$	$\{110\}$	2
			$l \frac{\infty P}{2}$	$\{1\bar{1}0\}$	
	Clino	$r, l\left[\frac{na' : \bar{b} : \infty \dot{c}}{2}\right]$	$r \frac{\infty P n'}{2}$	$\{hk0\}$	
			$l \frac{\infty P n'}{2}$	$\{h\bar{k}0\}$	
	Ortho	$r, l\left[\frac{a' : n\bar{b} : \infty \dot{c}}{2}\right]$	$r \frac{\infty P \bar{n}}{2}$	$\{kh0\}$	
			$l \frac{\infty P \bar{n}}{2}$	$\{k\bar{h}0\}$	
Hemi-clinodomes		$r, l\left[\frac{\infty a' : \bar{b} : m\dot{c}}{2}\right]$	$r \frac{mP\infty'}{2}$	$\{0hl\}$	2
			$l \frac{mP\infty'}{2}$	$\{0\bar{h}l\}$	
Clinopinacoids		$r, l\left[\frac{\infty a' : \bar{b} : \infty \dot{c}}{2}\right]$	$r \frac{\infty P\infty'}{2}$	$\{010\}$	1
			$l \frac{\infty P\infty'}{2}$	$\{0\bar{1}0\}$	
Hemi-orthodomes		$\pm a' : \infty \bar{b} : m\dot{c}$	Apparently holohedral		
Pina-coids	Basal	$\infty a' : \infty \bar{b} : \dot{c}$			
	Ortho	$a' : \infty \bar{b} : \infty \dot{c}$			

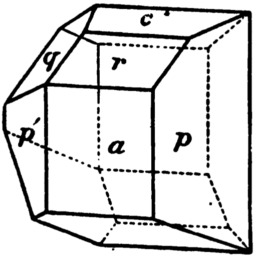

Fig. 390.

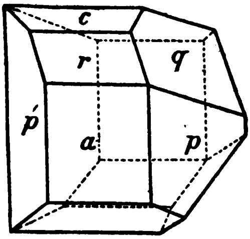

Fig. 391.

Combinations.

Figures 390 and 391. $a = \infty P \bar{\infty}$, $\{100\}$; $c =$ OP, $\{001\}$; $r = -P\bar{\infty}$, $\{101\}$; $p = r \frac{\infty P}{2}$, $\{110\}$; $p' = l \frac{\infty P}{2}$, $\{1\bar{1}0\}$; $q = l \frac{P\infty'}{2}$,

$\{0\bar{1}1\}$ (figure 390); $q = r\frac{P\infty'}{2}$, $\{011\}$ (figure 391). These combinations occur on tartaric acid, $C_4H_6O_6$. The above figures represent left- and right-handed crystals.

TRICLINIC SYSTEM.[1]

Crystallographic Axes. This system includes all forms which can be referred to three unequal axes intersecting each other at oblique angles. The axes are designated as in the orthorhombic system, namely, $\breve{a}$, brachydiagonal; $\bar{b}$, macrodiagonal; and $\acute{c}$, vertical axis. From this it follows that one axis must be held vertically, a second is directed toward the observer, and then the third is inclined from right to left or *vice versa.* Usually the brachydiagonal is the shorter of the two lateral axes. Figure 392 shows an axial cross of the triclinic system. The oblique angles between the axes are indicated as follows: $\bar{b} \wedge \acute{c} = \alpha$, $\breve{a} \wedge \acute{c} = \beta$, and $\breve{a} \wedge \bar{b} = \gamma$. The elements of crystallization consist of the axial ratio and the three angles α, β, and γ, page 8.

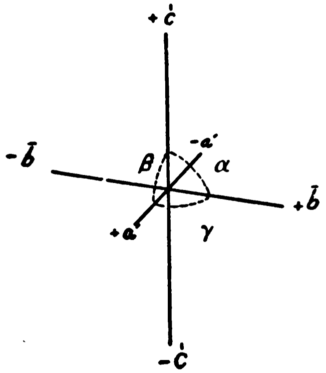

Fig. 392.

Classes of Symmetry. There are but two classes of symmetry in the triclinic system, namely:

1. Pinacoidal class (*Holohedrism*).
2. Asymmetric class (*Hemihedrism*).

The first is the important class.

1. PINACOIDAL CLASS.[2]

(*Holohedrism.*)

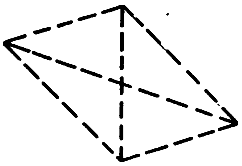

Fig. 393.

Symmetry. A center of symmetry is the only element present. Hence, forms can consist of but two faces, namely, face and parallel counter-face. This is represented diagrammatically by figure 393, which shows a triclinic combination projected upon the plane of the $\breve{a}$ and $\breve{b}$ axes.

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[1] Also termed the *anorthic*, *asymmetric* or *clinorhomboidal* system.

[2] *Normal group* of Dana.

Tetra-pyramids. As already shown, triclinic forms consist of but two faces. Therefore, since the planes of the crystallographic axes divide space into four pairs of dissimilar octants, it follows that four types of pyramidal forms must result. These are spoken of as *tetra-pyramids.*[1] There are, hence, four tetra-pyramids, each cutting the axes at their unit lengths. The same is also true of the *modified, brachy-* and *macro-bipyramids.* That is to say, the various bipyramids of the orthorhombic system, on account of the obliquity of the three axes, now yield four tetra-pyramids each. They are designated as *upper right, upper left, lower right,* and *lower left* forms, depending upon which of the front octants the form encloses. Naumann indicates them in turn by P′, ′P, P,, ,P. The general symbols for all types are given in the tabulation on page 141. Figure 394 shows the four unit tetra-pyramids in combination.

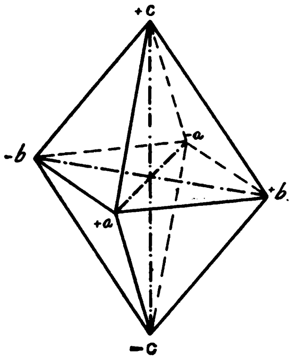

Fig. 394.

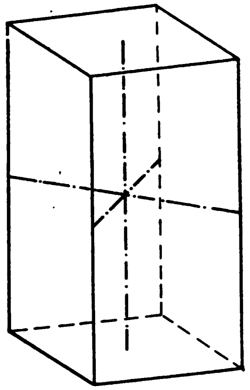

Fig. 395.

Hemi-prisms. Obviously the prisms are now to be designated as *right* and *left* forms. These two forms are in combination with the basal pinacoid in figure 395.

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1) In reality, *tetra-bipyramids*, see page 127.

Hemi-domes. All domes now consist of but two faces. Hence, we may speak of *right* and *left hemi-brachydomes*, and *upper* and *lower hemi-macrodomes*. These forms are shown in combination with the macro- and brachypinacoids, respectively, in figures 396 and 397.

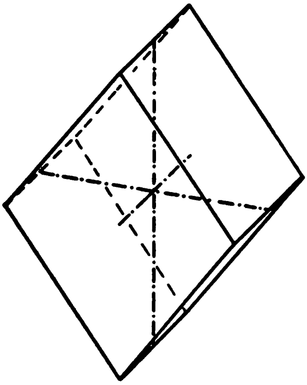

Fig. 396.

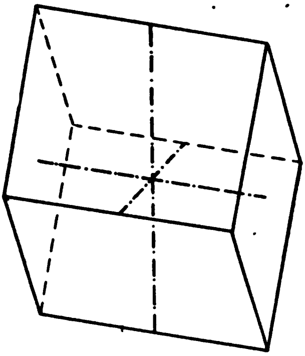

Fig. 397.

Pinacoids. These forms occur with their usual number of faces and are designated, as heretofore, by the terms *basal*, *brachy-*, and *macropinacoids*, depending upon the fact whether they extend parallel to the ć, ă, or b̄ axis. Figure 398 shows these pinacoids in combination.

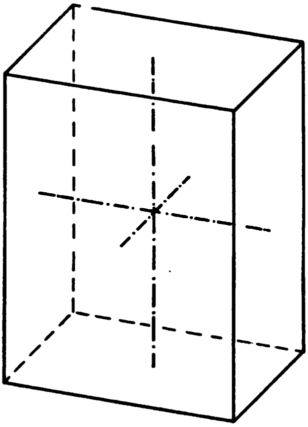

Fig. 398.

The various forms and symbols are given in the following table:

FORMS		SYMBOLS		
		Weiss	Naumann	Miller
Tetra-pyramids	Unit	$\breve{a} : \bar{b} : \dot{c}$	P'	$\{111\}$
		$\breve{a} : -\bar{b} : \dot{c}$	$'P$	$\{1\bar{1}1\}$
		$\breve{a} : \bar{b} : -\dot{c}$	$P_,$	$\{11\bar{1}\}$
		$\breve{a} : -\bar{b} : -\dot{c}$	$_,P$	$\{1\bar{1}\bar{1}\}$
	Modified	$\breve{a} : \bar{b} : m\dot{c}$	mP'	$\{hhl\}$
		$\breve{a} : -\bar{b} : m\dot{c}$	$m'P$	$\{h\bar{h}l\}$
		$\breve{a} : \bar{b} : -m\dot{c}$	$mP_,$	$\{hh\bar{l}\}$
		$\breve{a} : -\bar{b} : -m\dot{c}$	$m_,P$	$\{h\bar{h}\bar{l}\}$
	Brachy	$n\breve{a} : \bar{b} : m\dot{c}$	$mP'\breve{n}$	$\{hkl\}$
		$n\breve{a} : -\bar{b} : m\dot{c}$	$m'P\breve{n}$	$\{h\bar{k}l\}$
		$n\breve{a} : \bar{b} : -m\dot{c}$	$mP_,\breve{n}$	$\{hk\bar{l}\}$
		$n\breve{a} : -\bar{b} : -m\dot{c}$	$m_,P\breve{n}$	$\{h\bar{k}\bar{l}\}$
	Macro	$\breve{a} : n\bar{b} : m\dot{c}$	$mP'\bar{n}$	$\{khl\}$
		$\breve{a} : -n\bar{b} : m\dot{c}$	$m'P\bar{n}$	$\{k\bar{h}l\}$
		$\breve{a} : n\bar{b} : -m\dot{c}$	$mP_,\bar{n}$	$\{kh\bar{l}\}$
		$\breve{a} : -n\bar{b} : -m\dot{c}$	$m_,P\bar{n}$	$\{k\bar{h}\bar{l}\}$
Hemi-prisms	Unit	$\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P'_,$	$\{110\}$
		$\breve{a} : -\bar{b} : \infty\dot{c}$	$\infty{}'_,P$	$\{1\bar{1}0\}$
	Brachy	$n\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P'_,\breve{n}$	$\{hk0\}$
		$n\breve{a} : -\bar{b} : \infty\dot{c}$	$\infty{}'_,P\breve{n}$	$\{h\bar{k}0\}$
	Macro	$\breve{a} : n\bar{b} : \infty\dot{c}$	$\infty P'_,\bar{n}$	$\{kh0\}$
		$\breve{a} : -n\bar{b} : \infty\dot{c}$	$\infty{}'_,P\bar{n}$	$\{k\bar{h}0\}$
Hemi-domes	Brachy	$\infty\breve{a} : \bar{b} : m\dot{c}$	$m_,P'\breve{\infty}$	$\{0hl\}$
		$\infty\breve{a} : -\bar{b} : m\dot{c}$	$m'P_,\breve{\infty}$	$\{0\bar{h}l\}$
	Macro	$\breve{a} : \infty\bar{b} : m\dot{c}$	$m'P'\bar{\infty}$	$\{h0l\}$
		$\breve{a} : \infty\bar{b} : -m\dot{c}$	$m_,P_,\bar{\infty}$	$\{h0\bar{l}\}$
Pinacoids	Basal	$\infty\breve{a} : \infty\bar{b} : \dot{c}$	$0P$	$\{001\}$
	Brachy	$\infty\breve{a} : \bar{b} : \infty\dot{c}$	$\infty P\breve{\infty}$	$\{010\}$
	Macro	$\breve{a} : \infty\bar{b} : \infty\dot{c}$	$\infty P\bar{\infty}$	$\{100\}$

Relationship of forms. The diagram on page 120 may be used for this purpose, if we bear in mind that bipyramids, prisms, and domes, indicated in the same, now represent four tetra-pyramids, two hemi-prisms, and two hemi-domes, respectively.

Combinations. Figure 399. $x = P'\{111\}$; $r = {}'P\{1\bar{1}1\}$; $m = \infty P',\{110\}$; $M = \infty,'P\{1\bar{1}0\}$; $s = 2\,'P'\bar{\infty}\{201\}$; $a = \infty P\bar{\infty}\{100\}$. Axinite, $HCa_3Al_2BSi_4O_{16}$.

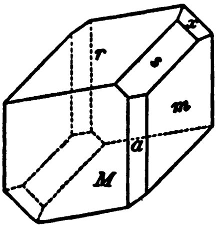

Fig. 399.

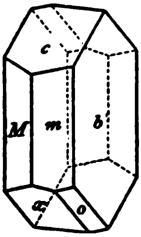

Fig. 400.

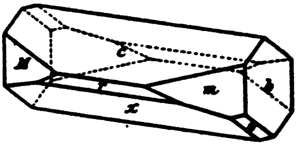

Fig. 401.

Figures 400 and 401. $m = \infty P',\{110\}$; $M = \infty,'P\{1\bar{1}0\}$; $b = \infty P\breve{\infty}\{010\}$; $c = 0P\{001\}$; $x = ,P,\infty\{10\bar{1}\}$; $o = P,\{11\bar{1}\}$; $r = \frac{4}{3},P,\infty\{40\bar{3}\}$. Albite, $NaAlSi_3O_8$.

2. *ASYMMETRIC CLASS.*1)

(*Hemihedrism.*)

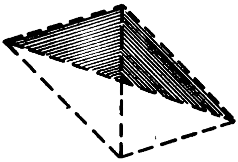

Fig. 402.

Symmetry. This class has no element of symmetry. This is clearly shown by figure 402.

Forms. Since the center of symmetry is lost, it follows that one face may constitute a form and, hence, this class possesses *ogdo-pyramids*, *tetra-prisms*, and *tetra-domes*. The forms are designated as in the preceding class, indicating, however, whether the octant in which the form lies is in front or not. For example,

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1) Dana calls this the *asymmetric group*.

$-\breve{a} : \bar{b} : \acute{c}$, $P'r$, $\{11\bar{1}\}$, is an *ogdopyramid* occupying the upper right rear octant, while $\breve{a} : \bar{b} : -\acute{c}$, P,f, $\{11\bar{1}\}$, is another in the lower right front octant.

The various forms and their symbols are easily deduced from those given on page 141.

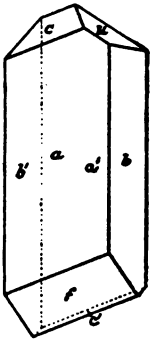

Fig. 403.

Combinations. Figure 403. $a = \infty P\bar{\infty}\ f$, $\{100\}$; $a' = \infty P\bar{\infty}\ r$, $\{\bar{1}00\}$; $b = \infty P\breve{\infty}\ r$, $\{010\}$; $b' = \infty P\breve{\infty}\ l$, $\{0\bar{1}0\}$; $c = 0Pu$, $\{001\}$; $\acute{c} = 0Pl$, $\{00\bar{1}\}$, $u = P'\breve{2}\ r$, $\{\bar{1}22\}$; $f = P\bar{\infty}l$, $\{10\bar{1}\}$; strontium bitartrate, $Sr(C_4H_4O_6H)_2.4H_2O$.

COMPOUND CRYSTALS.

Parallel Grouping. Oftentimes two crystals of the same substance are observed to have so intergrown that the crystallographic axes of the one individual are parallel to those of the other. Such an arrangement of crystals is termed *parallel grouping*. Figures 404 and 405[1] show such groups of quartz and calcite, respectively. Occasionally, crystals of different substances are observed grouped in this way.

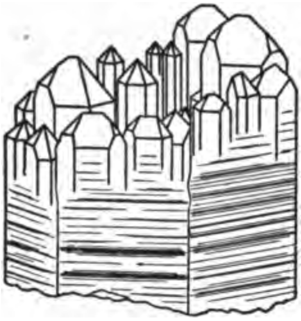
Fig. 404.

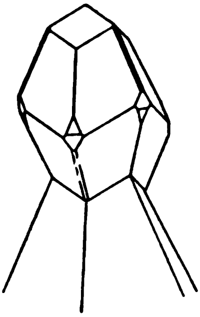
Fig. 405.

Twin Crystals. Two crystals may also intergrow so that, even though parallelism of the crystals is wanting, the growth has, nevertheless, taken place in some definite manner. Such crystals are spoken of as *twin crystals*, or in short, *twins*. Figure 406 illustrates such a twin crystal commonly observed on staurolite. In twin crystals both individuals have at least one crystal plane or a direction in common. Figure 407 shows a twinned octahedron. The plane common to both parts is termed the *composition plane*. In general, the plane to which the twin crystal is symmetrical is the *twinning plane*. In some instances, *composition* and *twinning planes* coincide. Both, however, are parallel to some possible face of the

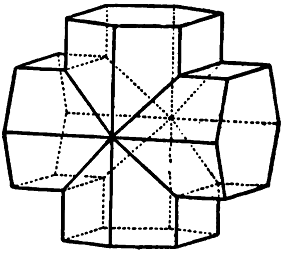
Fig. 406.

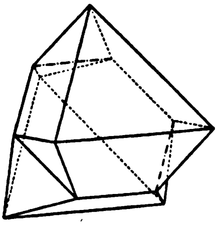
Fig. 407.

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[1] After Tschermak and Melczer, respectively.

crystal, which is not parallel to a plane of symmetry. The line or direction perpendicular to the twinning plane is the *twinning axis.* A *twinning law* is expressed by indicating the *twinning plane* or *axis*.

Twin crystals are commonly divided into two classes: 1) *Contact* or *Juxtaposition* twins, and 2) *Penetration* twins.[1] These are illustrated by figures 407 and 406, respectively. Contact twins consist of two individuals so placed that if one be rotated through 180° about the twinning axis the simple crystal results. In penetration twins two individuals have interpenetrated one another. If one of the individuals be rotated through 180° about the twinning axis both individuals will occupy the same position.

Contact and penetration twins are comparatively common in all systems. In studying twins, it must be borne in mind, as pointed out on page 3, that the two individuals may not be symmetrical owing to distortion. Re-entrant angles are commonly indicative of twinning.

Common twinning laws. Cubic System. The most common law in the cubic system is that known as the *spinel* law, where the twinning plane is parallel to a face of an octahedron, O{111}. Figure 407 shows such a twin crystal of the mineral spinel, while figure 408 shows a twinned octahedron of magnetite. A penetration twin of fluorite is shown in figure 409. Here, two cubes interpenetrate according to the above law. Figure 410 shows a penetration twin of two tetrahedrons, observed on the diamond. The twinning plane is a plane parallel to a face of a cube, ∞O∞{100}, which in

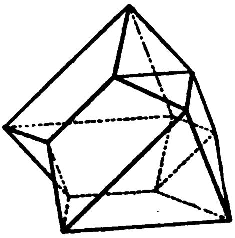
Fig. 408.

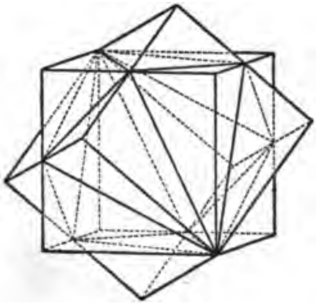
Fig. 409.

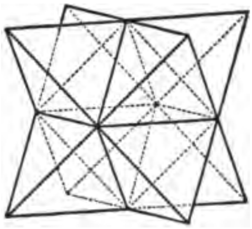
Fig. 410.

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[1] Also designated as *reflection* and *rotation* twins, because they are symmetrical to a plane and an axis, respectively.

the hextetrahedral class is no longer a plane of symmetry. The axes are parallel in the two individuals and since the twinning tends to give the twin the symmetry of the hexoctahedral class, such twinned crystals are sometimes spoken of as *supplementary twins*.

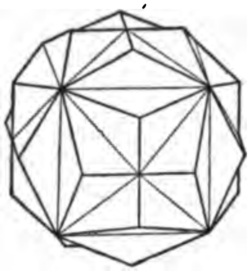

Fig. 411.

Of the minerals crystallizing in the dyakisdodecahedral class, pyrite furnishes excellent twins. Figure 411 shows a penetration twin of two pyritohedrons of this mineral. Such twins are often known as crystals of the *iron cross*. A plane parallel to a face of the rhombic dodecahedron, ∞O$\{110\}$, is the twinning plane. Such a plane is parallel to the secondary planes of symmetry which are not present in this class.

Hexagonal System. Calcite and quartz are the only common minerals belonging to this system which furnish good examples of twinning. These minerals belong to the ditrigonal scalenohedral and trigonal trapezohedral classes, respectively.

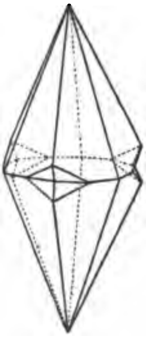

Fig. 412.

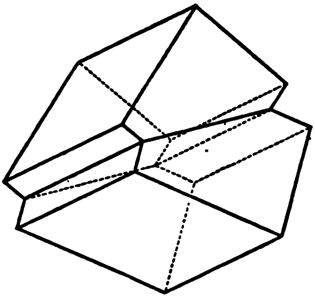

Fig. 413.

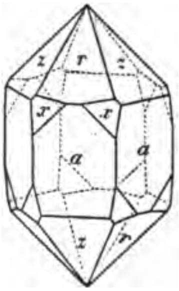

Fig. 414.

Upon calcite the basal pinacoid, OR$\{0001\}$, is commonly a twinning plane. Figure 412 illustrates this law.[1] A plane parallel to a face of the negative rhombohedron, $-\frac{1}{2}R\{01\bar{1}2\}$, may also be a twinning plane as illustrated by figure 413. These are the most common laws on calcite although crystals possessing a twinning plane parallel to a face of the rhombohedron, $-2R\{02\bar{2}1\}$, are also to be noted.

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1) Compare with figure 187.

The so-called Brazilian law is common on twins of quartz, figure 414. Here, a right and left crystal have interpenetrated so that the twin is now symmetrical to a plane parallel to a face of the prism of the second order, $\infty P2\ \{11\bar{2}0\}$, compare figures 253 and 254. Figure 414, as is obvious, possesses a higher grade of symmetry than its component individuals, namely, that of the ditrigonal scalenohedral class.

Tetragonal System. Most of the twin crystals of this system are to be observed on substances crystallizing in the ditetragonal bipyramidal class. In this class a plane parallel to a face of the unit bipyramid of the second order, $P\infty\ \{011\}$, commonly acts as the twinning plane. Figures 415 and 416 show crystals of cassiterite and zircon, respectively, twinned according to this law.

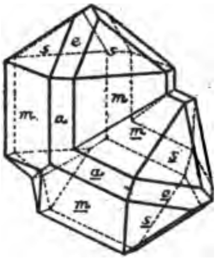

Fig. 415.

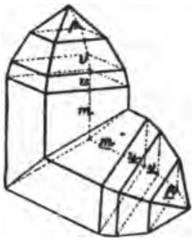

Fig. 416.

Orthorhombic System. The most common twins of this system belong to the bipyramidal (holohedral) class in which any face aside from the pinacoids may act as twinning plane. Figure 406, page 144, shows a penetration twin of staurolite, where the brachydome, $\frac{3}{2}\breve{P}\breve{\infty}$ $\{032\}$, acts as the twinning plane. Figure 417 shows the same mineral with the pyramid, $\frac{3}{2}\breve{P}\frac{3}{2}$ $\{232\}$, as the twinning plane. Figure 418 represents a contact twin of aragonite. Here the unit prism, ∞P $\{110\}$, is the twinning plane.

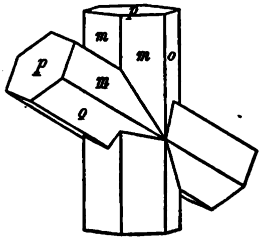

Fig. 417.

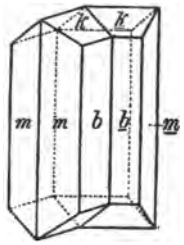

Fig. 418.

Monoclinic System. In this system, gypsum and orthoclase furnish some of the best examples. Figure 419 shows a contact twin of gypsum in which the orthopinacoid, $\infty\bar{P}\bar{\infty}\ \{100\}$, is the twinning plane. A penetration twin of orthoclase is shown in figure

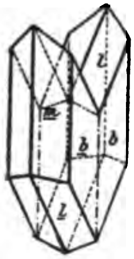

Fig. 419.

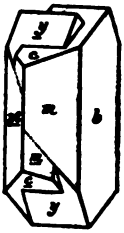

Fig. 420.

420. Here, the orthopinacoid also acts as twinning plane. This is known as the *Karlsbad law* on orthoclase.

Triclinic System. Since there are no planes of symmetry in this system, any plane may act as the twinning plane. The mineral albite furnishes good examples. In figure 421, the clinopinacoid, $\infty P\breve{\infty}$ $\{010\}$, acts as the twinning plane. This is the *albite law*. Another common law is shown by figure 422. Here, the basal pinacoids of both individuals are parallel, the crystallographic $\bar{b}$ axis acting as the twinning axis. This is known as the *pericline law*.

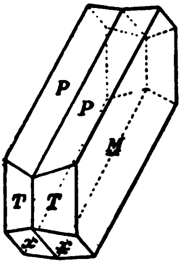

Fig. 421.

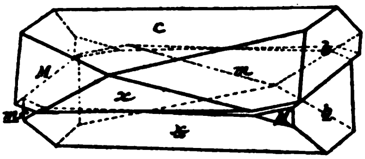

Fig. 422.

Repeated twinning. In the foregoing, crystals consisting of but two individuals have been discussed. Intergrowths of three, four, five, etc., individuals are termed threelings, fourlings, fivelings, and so on. *Polysynthetic* and *cyclic* twins are the result of repeated twinning. In the polysynthetic twins the twinning planes between any two individuals are parallel. This is illustrated by figures 423 and 424 showing polysynthetic twins of albite and aragonite, respectively.[1] If the individuals are very thin the re-entrant angles are usually

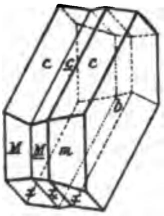

Fig. 423.

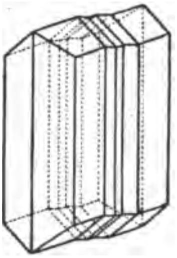

Fig. 424.

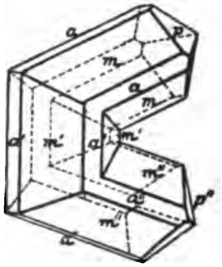

Fig. 425.

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[1] Compare with figures 418 and 421.

indicated by striations. The cyclic twins result when the twinning planes do not remain parallel, as for example when adjacent faces of a form act as twinning planes. This is shown by the cyclic twins of rutile, figure 425, in which adjoining faces of the unit bipyramid of the second order $\infty P2 \{011\}$ act as twinning planes.

Mimicry. As a result of repeated twinning, forms of an apparently higher grade of symmetry usually result. This is especially common with those substances possessing pseudosymmetry, page 94. Figure 426 shows a trilling of the orthorhombic mineral aragonite, $CaCO_3$, which is now apparently hexagonal in its outline. In figure 427 the cross-section is shown. This phenomenon is called *mimicry*.

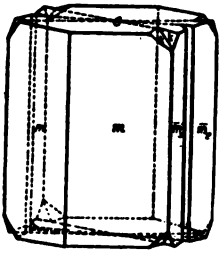

Fig. 426.

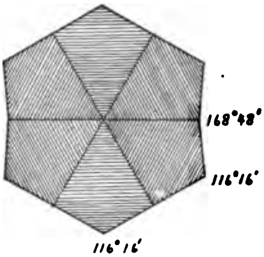

Fig. 427.

TABULAR CLASSIFICATION
SHOWING THE ELEMENTS OF SYMMETRY
AND THE SIMPLE FORMS
OF THE THIRTY-TWO CLASSES OF CRYSTALS.

CUBIC SYSTEM.

Classes of Crystals, 1 to 5.

(Page 150.)

TABULAR CLASSIFICATION SHOWING THE ELEMENTS OF SYMMETRY A

1. CUBIC

CLASS	SYMMETRY						a : a : a O {111}	a : a : ∞a ∞O {110}	a :
	Planes		Axes			Center			
	Principal	Secondary	■	▲	●				
1. **Hexoctahedral** (*Holohedrism*)	3	6	3	4	6	1	Octahedron	Rhombic Dodecahedron	He
2. **Hextetrahedral** (*Tetrahedral Hemihedrism*)	—	6	—	4 (*Polar*)	3	—	Tetrahedrons (±)		
3. **Dyakisdodecahedral** (*Pyritohedral Hemihedrism*)	3	—	—	4	3	1			
4. **Pentagonal Icositetrahedral** (*Plagihedral Hemihedrism*)	—	—	3	4	6	—			
5. **Tetrahedral Pentagonal Dodecahedral** (*Tetartohedrism*)	—	—	—	4 (*Polar*)	3	—	Tetrahedrons (±)		

1) The blank spaces indicate that the forms are *apparently* holohedral.

AND THE SIMPLE FORMS OF THE THIRTY-TWO CLASSES OF CRYSTALS.

C SYSTEM.1)

FORMS					REPRESENTATIVE
: ∞a : ∞a $\infty O \infty$ $\{100\}$	$a : a : ma$ mO $\{hhl\}$	$a : ma : ma$ mOm $\{hll\}$	$a : ma : \infty a$ ∞Om $\{hko\}$	$a : na : ma$ mOn $\{hkl\}$	
Hexahedron 6	Trigonal Trisoctahedron 24	Tetragonal Trisoctahedron 24	Tetrahexahedron 24	Hexoctahedron 4	Galena (PbS)
	Tetragonal Tristetrahedrons (±)	Trigonal Tristetrahedrons (±)		Hextetrahedrons (±)	Tetrahedrite $(Cu_2, Fe, Zn)_4$ $(AS, Sb)_2S_7$
			Pyritohedrons (±)	Dyakisdodecahedrons (±)	Pyrite (FeS_2)
				Pentagonal Icositetrahedrons (*r*, *l*)	Sal Ammoniac (NH_4Cl)
	Tetragonal Tristetrahedrons (±)	Trigonal Tristetrahedrons (±)	Pyritohedrons (±)	Tetrahedral Pentagonal Dodecahedrons ±*r*, ±*l*	Sodium Chlorate $(NaClO_3)$

HEXAGONAL SYSTEM.

Classes of Crystals, 6 to 11.

(Page 151.)

CLASS	SYMMETRY									
	Planes			Axes				$u : \infty a : a : mc$	$2a : 2a : a : mc$	$na :$
	Principal	Secondary	Intermediat	⬢	▲	⬬	Center	mP $\{10\bar{1}1\}$	mP2 $\{h\,h\,\overline{2h}\,l\}$	$\{h$
6. **Dihexagonal Bipyramidal** (*Holohedrism*)	1	3	3	1	–	3+3	1	Hexagonal Bipyramids First order	Hexagonal Bipyramids Second order	Dihe Bip
7. **Dihexagonal Pyramidal** [*Holohedrism with Hemimorphism*]	–	3	3	1 (*Polar*)	–	–	–	Hexagonal Pyramids First order (*u*, *l*)	Hexagonal Pyramids Second order (*u*, *l*)	Dih Py (
8. **Ditrigonal Bipyramidal** [*Trigonal Hemihedrism*]	1	–	3	–	1	3 (*Polar*)	–	Trigonal Bipyramids First order (±)		Dit Bip (
9. **Ditrigonal Scalenohedral** [*Rhombohedral Hemihedrism*]	–	–	3	–	1	3	1	Rhombohedrons First order (±)		Sca c (
10. **Hexagonal Bipyramidal** [*Pyramidal Hemihedrism*]	1	–	–	1	–	–	1			He Bip Thi (
11. **Hexagonal Trapezohedral** [*Trapezohedral Hemihedrism*]	–	–	–	1		3+3	–			He Tra c (

FORMS					
$a:pa:a:mc$ mPn $\{hi\bar{k}l\}$	$a:\infty a:a:\infty c$ ∞P $\{10\bar{1}0\}$	$2a:2a:a:\infty c$ $\infty P2$ $\{11\bar{2}0\}$	$na:pa:a:\infty c$ ∞Pn $\{hi\bar{k}0\}$	$\infty a:\infty a:\infty a:c$ OP $\{0001\}$	**REPRESENTATIVE**
ihexagonal Bipyramids	Hexagonal Prism First order	Hexagonal Prism Second order	Dihexagonal Prisms	Basal Pinacoid	**Beryl ($Be_3Al_2(SiO_3)_6$)**
Dihexagonal Pyramids (*u*, *l*)				Basal Pinacoids (*u*, *l*)	**Zincite (ZnO)**
Ditrigonal Bipyramids (±)	Trigonal Prisms First order (±)		Ditrigonal Prisms (±)		——
Scalenohedrons (±)					**Calcite ($CaCO_3$)**
Hexagonal Bipyramids Third order (±)			Hexagonal Prisms Third order (±)		**Apatite ($Ca_5Cl(PO_4)_3$)**
Hexagonal Trapezohedrons (*r*, *l*)					**Barium Stibiotartrate and Potassium Nitrate $\left[\begin{matrix} Ba\ (SbO)_2 \\ C_4H_4O_6)_2+ \\ KNO_3 \end{matrix}\right]$**

HEXAGONAL SYSTEM.

(*Continued*)

Classes of Crystals, 12 to 17.

(Page 152.)

CLASS	SYMMETRY								
	Planes			Axes				$a:\infty a:a:mc$ *m*P $\{10\bar{1}1\}$	$2a:2a:a:mc$ *m*P2 $\{h\,h\,\overline{2h}\,l\}$	$na:$ {
	Principal	Secondary	Intermediate	⬢	▲	●	Center			
12. **Ditrigonal Pyramidal** [*Trigonal Hemihedrism with Hemimorphism*]	–	–	3	–	1 (*Polar*)	–	–	Trigonal Pyramids First order $(\pm u, \pm l)$	Hexagonal Pyramids Second order (u, l)	D P (
13. **Hexagonal Pyramidal** [*Pyramidal Hemihedrism with Hemimorphism*]	–	–	–	1 (*Polar*)	–	–	–	Hexagonal Pyramids First order (u, l)	Hexagonal Pyramids Second order (u, l)	He P Th (
14. **Trigonal Bipyramidal** [*Trigonal Tetartohedrism*]	1	–	–	–	1	–	–	Trigonal Bipyramids First order (±)	Trigonal Bipyramids Second order (±)	T Bi Th
15. **Trigonal Trapezohedral** [*Trapezohedral Tetartohedrism*]	–	–	–	–	1	3 (*Polar*)	–	Rhombohedrons First order (±)	Trigonal Bipyramids Second order (±)	T Tr (
16. **Trigonal Rhombohedral** [*Rhombohedral Tetartohedrism*]	–	–	–	–	1	–	1	Rhombohedrons First order (±)	Rhombohedrons Second order (±)	Rh Th ±
17. **Trigonal Pyramidal** (*Ogdohedrism*)	–	–	–	–	1 (*Polar*)	–	–	Trigonal Pyramids First order $(\pm u, \pm l)$	Trigonal Pyramids Second order $(\pm u, \pm l)$	T P Th

Continued)

FORMS					
$na:pa:a:mc$ mPn $\{hi\bar{k}l\}$	$a:\infty a:a:\infty c$ ∞P $\{10\bar{1}0\}$	$2a:2a:a:\infty c$ $\infty P2$ $\{11\bar{2}0\}$	$na:pa:a:\infty c$ ∞Pn $\{hi\bar{k}o\}$	$\infty a:\infty a:\infty a:c$ OP $\{0001\}$	REPRESENTATIVE
Ditrigonal Pyramids (±u, ±l)	Trigonal Prisms First order (±)		Ditrigonal Prisms (±)	Basal Pinacoids (u, l)	Tourmaline $Al_3B_2O_5H_2\left\{\frac{Al}{3},\frac{Mg}{2},\frac{Fe}{2}, Li, Na, H\right\}_9(SiO_4)_4$
Hexagonal Pyramids Third order (±u, ±l)			Hexagonal Prisms Third order (±)	Basal Pinacoid u, l	Strontium Antimonyl-tartrate $Sr(SbO)_2(C_4H_4O_6)_2$
Trigonal Bipyramids Third order ±r, ±l	Trigonal Prisms First order (±)	Trigonal Prisms Second order (±)	Trigonal Prisms Third order (±r, ±l)		———
Trigonal Trapezohedrons (±r, ±l)		Trigonal Prisms Second order (±)	Ditrigonal Prisms (±)		Quartz (SiO_2)
Rhombohedrons Third order ±r, ±l			Hexagonal Prisms Third order (±)		Dioptase (H_2CuSiO_4)
Trigonal Pyramids Third order ±r, u; ±r, l; ±l, u; ±l, l;	Trigonal Prisms First order (±)	Trigonal Prisms Second order (±)	Trigonal Prisms Third order (±r, ±l)	Basal Pinacoids (u, l)	Sodium Periodate ($NaIO_4+3H_2O$)

TETRAGONAL SYSTEM.

Classes of Crystals, 18 to 24.

(Page 153.)

CLASS	SYMMETRY						$a : a : mc$ mP $\{h\,h\,l\}$	$a : \infty a : mc$ mP∞ $\{h\,o\,l\}$
	Planes			Axes		Center		
	Principal	Secondary	Intermediate	■	●			
18. **Ditetragonal Bipyramidal** (*Holohedrism*)	1	2	2	1	2 + 2	1	Tetragonal Bipyramids First order	Tetragonal Bipyramids Second order
19. **Ditetragonal Pyramidal** [*Holohedrism with Hemimorphism*]	—	2	2	1 (*Polar*)	—	—	Tetragonal Pyramids First order (*u*, *l*)	Tetragonal Pyramids Second order (*u*, *l*)
20. **Tetragonal Scalenohedral** [*Sphenoidal Hemihedrism*]	—	—	2	—	1 + 2	—	Tetragonal Bisphenoids First order (±)	
21. **Tetragonal Bipyramidal** [*Pyramidal Hemihedrism*]	1	—	—	1	—	1		
22. **Tetragonal Trapezohedral** [*Trapezohedral Hemihedrism*]	—	—	—	1	2 + 2	—		
23. **Tetragonal Pyramidal** [*Pyramidal Hemihedrism with Hemimorphism*]	—	—	—	1 (*Polar*)	—	—	Tetragonal Pyramids First order (*u*, *l*)	Tetragonal Pyramids Second order (*u*, *l*)
24. **Tetragonal Bisphenoidal** (*Tetartohedrism*)	—	—	—	—	1	—	Tetragonal Bisphenoids First order (±)	Tetragonal Bisphenoids Second order (±)

FORMS					REPRESENTATIVE
$a : na : mc$ mPn $\{hkl\}$	$a : a : \infty c$ ∞P $\{110\}$	$a : \infty a : \infty c$ $\infty P \infty$ $\{100\}$	$a : na : \infty c$ ∞Pn $\{hko\}$	$\infty a : \infty a : c$ OP $\{001\}$	
Ditetragonal Bipyramids	Tetragonal Prism First order	Tetragonal Prism Second order	Ditetragonal Prisms	Basal Pinacoid	Cassiterite (SnO_2)
Ditetragonal Pyramids (*u*, *l*)				Basal Pinacoids (*u*, *l*)	Silver Fluoride ($AgF + H_2O$)
Tetragonal Scalenohedrons (±)					Chalcopyrite ($CuFeS_2$)
Tetragonal Bipyramids Third order (±)			Tetragonal Prisms Third order (±)		Scheelite ($CaWO_4$)
Tetragonal Trapezohedrons (*r*, *l*)					Nickel Sulphate ($NiSO_4 + 6H_2O$)
Tetragonal Pyramids Third order ±*u*, ±*l*			Tetragonal Prisms Third order (±)	Basal Pinacoids (*u*, *l*)	Wulfenite ($PbMoO_4$)
Tetragonal Bisphenoids Third order ±*r*, ±*l*			Tetragonal Prisms Third order (±)		——

ORTHORHOMBIC SYSTEM.

Classes of Crystals, 25 to 27.

(Page 154.)

CLASS	SYMMETRY					
	Planes	● Axes	Center	$n\breve{a} : \bar{b} : m\dot{c}$ $mP\breve{n}$ $\{hkl\}$	$n\breve{a} : \bar{b} : \infty\dot{c}$ $\infty P\breve{n}$ $\{hk0\}$	$\infty\breve{a}$: m $\{0$
25. **Orthorhombic Bipyramidal** (*Holohedrism*)	1 + 1 + 1	1 + 1 + 1	1	Orthorhombic Bipyramids	Orthorhombic Prisms	Brach
26. **Orthorhombic Pyramidal** [*Holohedrism with Hemimorphism*]	1 + 1	1 (*Polar*)	—	Orthorhombic Pyramids (u, l)		Brach (
27. **Orthorhombic Bisphenoidal** (*Hemihedrism*)	--	1 + 1 + 1	—	Orthorhombic Bisphenoids (r, l)		

FORMS					REPRESENTATIVE
$\bar{b} : m\dot{c}$ $\mathrm{P}\breve{\infty}$ $h\,l\}$	$\breve{a} : \infty\bar{b} : m\dot{c}$ $m\mathrm{P}\bar{\infty}$ $\{h\,o\,l\}$	$\infty\breve{a} : \bar{b} : \infty\dot{c}$ $\infty\mathrm{P}\breve{\infty}$ $\{010\}$	$\breve{a} : \infty\bar{b} : \infty\dot{c}$ $\infty\mathrm{P}\bar{\infty}$ $\{100\}$	$\infty\breve{a} : \infty\bar{b} : \dot{c}$ OP $\{001\}$	
ydomes	Macrodomes	Brachy-pinacoid	Macro-pinacoid	Basal Pinacoid	Barite ($BaSO_4$)
ydomes l)	Macrodomes (u, l)			Basal Pinacoids (u, l)	Calamine ($Zn_2(OH)_2SiO_3$)
					Epsomite ($MgSO_4+7H_2O$)

MONOCLINIC SYSTEM.

Classes of Crystals, 28 to 30.

(Page 155.)

CLASS	SYMMETRY					
	Plane	● Axis	Center	$na' : \bar{b} : m\grave{c}$ mPn' $\{h\,k\,l\}$	$na' : \bar{b} : \infty\grave{c}$ $\infty Pn'$ $\{h\,k\,0\}$	$\infty a' : \bar{b} : m\grave{c}$ $mP\infty'$ $\{0\,h\,l\}$
28. **Monoclinic Prismatic** (*Holohedrism*)	1	1	1	Hemi-pyramids (±)	Prisms	Clinodomes
29. **Monoclinic Domatic** (*Hemihedrism*)	1	—	—	Tetra-pyramids (±*u*, ±*l*)	Hemi-prisms (*f*, *r*)	Hemi-clinodomes (*u*, *l*)
30. **Monoclinic Sphenoidal** (*Hemimorphism*)	—	1 (*Polar*)	—	Tetra-pyramids (*Sphenoids*) (±*r*, ±*l*)	Hemi-prisms (*r*, *l*)	Hemi-clinodomes (*r*, *l*)

FORMS				REPRESENTATIVE
$' : \infty \bar{b} : m\dot{c}$ $m\mathrm{P}\bar{\infty}$ $\{h0l\}$	$\infty a' : \bar{b} : \infty \dot{c}$ $\infty \mathrm{P} \infty'$ $\{010\}$	$a' : \infty \bar{b} : \infty \dot{c}$ $\infty \mathrm{P} \bar{\infty}$ $\{100\}$	$\infty a' : \infty \bar{b} : \dot{c}$ OP $\{001\}$	
Hemi-orthodomes ($\pm$)	Clinopinacoid	Orthopinacoid	Basal Pinacoid	Gypsum ($CaSO_4 + 2H_2O$)
Tetra-orthodomes ($\pm u$, $\pm l$)		Orthopinacoids (f, r)	Basal Pinacoids (u, l)	Tetrathionate of Potassium ($K_2S_4O_6$)
	Clinopinacoids (r, l)			Tartaric Acid ($C_4H_6O_6$)

TRICLINIC SYSTEM.

Classes of Crystals, 31 and 32.

(Page 156.)

CLASS	SYMMETRY						
	Planes	Axis	Center	$n\breve{a} : \bar{b} : m\dot{c}$ $mP\breve{n}$ $\{h\,k\,l\}$	$n\breve{a} : \bar{b} : \infty\dot{c}$ $\infty P\breve{n}$ $\{h\,k\,o\}$	$\infty a' : \bar{b} : m\dot{c}$ $mP\infty'$ $\{o\,h\,l\}$	$\breve{a}$
31. **Triclinic Pinacoidal** (*Holohedrism*)	—	—	I	Tetra-pyramids $\left[\frac{u}{l}r, \frac{u}{l}l\right]$	Hemi-prisms (r, l)	Hemi-brachydomes (r, l)	m
32. **Triclinic Asymmetric** (*Hemihedrism*)	—	—	—	Ogdo-pyramids $\left[\begin{matrix}\frac{u}{l}r, f; \\ \frac{u}{l}l, f; \\ \frac{u}{l}r, r; \\ \frac{u}{l}l, r;\end{matrix}\right]$	Tetra-prisms $\left[\frac{r}{l}f, \frac{r}{l}r\right]$	Tetra-brachydomes $\left[\frac{u}{l}r, \frac{u}{l}l\right]$	m

FORMS				REPRESENTATIVE
$: \infty \bar{b} : m\dot{c}$ $mP\bar{\infty}$ $\{h\,o\,l\}$	$\infty \breve{a} : \bar{b} : \infty \dot{c}$ $\infty P\breve{\infty}$ $\{010\}$	$\breve{a} : \infty \bar{b} : \infty \dot{c}$ $\infty P\bar{\infty}$ $\{100\}$	$\breve{a} : \infty \bar{b} : \infty \dot{c}$ OP $\{001\}$	
Hemi-macrodomes (u, l)	Brachypinacoid	Macropinacoid	Basal Pinacoid	Albite ($NaAlSi_3O_8$)
Tetra-macrodomes $\left[\frac{u}{l}f, \frac{u}{l}r\right]$	Brachypinacoids (l, r)	Macropinacoids (f, r)	Basal Pinacoids (u, l)	Strontium bitartrate $\left[\begin{matrix} Sr(C_4H_4O_6H)_2 + \\ 4H_2O \end{matrix}\right]$

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